System Identification Toolbox

For Use with MATLAB®

Lennart Ljung

- Computation
- Visualization
- Programming

User's Guide

Version 6

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Getting Started

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What Is the System Identification Toolbox?

The System Identification Toolbox enables you to build accurate, simplified models of complex systems from noisy time-series data.

It provides tools for creating mathematical models of dynamic systems based on observed input/output data. The toolbox features a flexible graphical user interface that aids in the organization of data and models. The identification techniques provided with this toolbox are useful for applications ranging from control system design and signal processing to time-series analysis and vibration analysis.

For Simulink® users, the System Identification Toolbox provides a library, `slident`, that contains blocks for performing system identification in the Simulink block diagram environment. In addition, you can use this library to do the following:

- Simulate any `idmodel` with or without noise
- Use `iddata` objects as data sources and sinks

Basic Questions About System Identification

What is system identification?

System identification enables you to build mathematical models of a dynamic system based on measured data. You adjust the parameters of a given model until its output coincides as well as possible with the measured output.

How do you know if the model is any good?

A good test is to compare the output of the model to measured data that was not used for the fit (called validation data).

Can the quality of the model be tested in other ways?

It is also valuable to look at the data that could not be reproduced by the model (the residuals). This should not be correlated with other available information, such as the system's input.

What models are most common?

The techniques apply to general models. The most common models are difference-equation descriptions, such as ARX and ARMAX models, as well as all types of linear state-space models.

Do you have to assume a model of a particular type?

For parametric models, you specify the model structure. This can be as easy as selecting a single integer — the model order — or it can involve several choices. If you assume that the system is linear, you can directly estimate its impulse or step response by using correlation analysis, or its frequency response by using spectral analysis. This enables useful comparisons with other estimated models.

What does the System Identification Toolbox contain?

It contains all the common techniques used to adjust parameters in all kinds of linear models. It also enables you to examine the quality of model properties, as well as to preprocess and polish the measured data.

Isn't it a big limitation to work only with linear models?

No, actually not. Many common model nonlinearities are such that the measured data should be nonlinearly transformed (by squaring a voltage input if the stimulus is the power, for example). You can get quite far by using

physical insight about the system you are modeling to determine the appropriate transformations of variables that may make the model linear.

How do I get started?

If you are a beginner, browse through Chapter 2, "The Graphical User Interface." Then try out a couple of the data sets that come with the toolbox. Use the graphical user interface (GUI) and check out the built-in help functions.

Is this really all there is to system identification?

There is a great deal written on the subject of system identification. However, the best way to explore system identification is by working with real data. It is important to remember that any estimated model, no matter how good it looks on your screen, is only a simplified reflection of reality. Surprisingly often, this is sufficient for rational decision-making.

Common Terms Used in System Identification

This section defines some of the terms that are frequently used in system identification:

- **Estimation data** is the data set that is used to create a model of the data. In the GUI, this is the same as *working data*.
- **Validation data** is the data set (different from estimation data) that is used to validate the model. Validation is accomplished by simulating the model for the validation data and then computing the residuals from the model for this data.
- **Model views** are the various ways of inspecting the properties of a model, such as zeros and poles, as well as transient and frequency responses.
- **Data views** are the various ways of inspecting the properties of data sets. It is most common and useful to plot the data and scrutinize it for so-called *outliers*. These are unreliable measurements that can arise from failures in the measurement equipment. Furthermore, the frequency content of the data signals can also be most revealing when viewed on a periodogram or a spectral estimate.
- **Model sets** or **model structures** are families of models with adjustable parameters. *Parameter estimation* is the process of finding the "best" values of these adjustable parameters. The system identification problem is to find both the model structure and good numerical values of the model parameters.
- **Parametric identification methods** are techniques for estimating parameters for a given model structure. This is a matter of using numerical search to find those numerical values of the parameters that give the best agreement between the model's (simulated or predicted) output and the measured output.
- **Nonparametric identification methods** are techniques to estimate model behavior without necessarily using a given parameterized model set. Typical nonparametric methods include **correlation analysis**, which estimates a system's impulse response, and **spectral analysis**, which estimates a system's frequency response.

- **Model validation** is the process of gaining confidence in a model. This is a highly subjective task, which involves scrutinizing all aspects of the model properties. An important tool is to study the model's ability to reproduce the behavior of the validation data set by simulation and prediction. Another useful technique is to analyze the properties of the residuals.

Basic Information About Dynamic Models

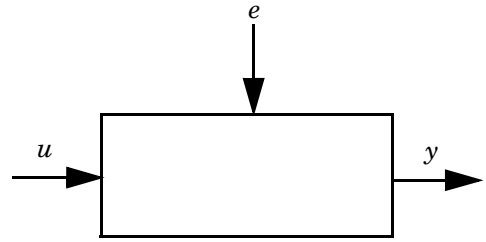
System identification is about building *dynamic models*. Therefore, some knowledge about dynamic models is a prerequisite for using this toolbox successfully. This topic is addressed in several places in Chapter 3, "Tutorial." Numerous textbooks are also available for introductory and in-depth study. This section describes what you need to know about dynamic models at the most basic level to get started with the System Identification Toolbox.

Signals

Models describe relationships between measured signals. It is convenient to distinguish between *input* signals and *output* signals, such that the outputs are partly determined by the inputs.

For example, consider an airplane where the inputs are control surfaces, such as ailerons and elevators, and the outputs are the orientation, velocity, and position of the airplane. In most cases, the outputs are also affected by signals other than the measured inputs. Such unmeasured inputs are called *disturbance* signals or *noise*. For the airplane, these additional signals would be wind gusts and turbulence effects.

If inputs, outputs, and disturbances are denoted by u, y, and e, respectively, the relationship is depicted in the following figure.



Input Signals *u*, Output Signals *y*, and Disturbances *e*

All these signals are functions of time, and the value of the input at time t is denoted by $u(t)$. The modeling problem is to describe how these three signals are related. In system identification, only discrete-time points are often considered because instruments typically record signals at discrete time instants, which are typically equally spaced with a *sampling interval* of T time units.

The Basic Dynamic Model

The basic relationship between signals is the *linear difference equation*. For example, consider the equation

$$y(t) - 1.5y(t-T) + 0.7y(t-2T) = 0.9u(t-2T) + 0.5u(t-3T) \qquad (ARX)$$

Such a relationship informs us how to compute the output *y(t)* if the input is known and the disturbance can be ignored:

$$y(t) = 1.5y(t-T) - 0.7y(t-2T) + 0.9u(t-2T) + 0.5u(t-3T)$$

Therefore, the output at time t is a linear combination of past outputs and past inputs. This is a *dynamic* model because the output at time t depends on the input signal at previous time instants.

In this case, the system identification problem is then to use measurements of u and y to determine

- The coefficients (such as -1.5 and 0.7)
- How many delayed outputs to use in the description (in this example, there are two: *y(t-T)* and *y(t-2T)*)
- The *time delay* in the system (in the second equation, the time delay is $2T$ because it takes $2T$ time units before a change in u affects y).
- How many delayed inputs to use (two in the example: *u(t-2T)* and *u(t-3T)*). The number of delayed inputs and outputs is usually referred to as the *model order*.

Variants of Model Descriptions

The basic dynamic model given above is called an *ARX* model. There are several variants of this model known as *output-error* (OE) models, *ARMAX* models, *FIR* models, and *Box-Jenkins* (BJ) models, which are described later in this book. At a basic level, it is sufficient to think of these models as variants of the ARX model that also include a characterization of the properties of the disturbance e.

Linear state- space models are another class of models, which is treated in more detail below. The essential model-structure variable is the model order, which is a scalar. Then, you only have “one knob to turn” when you search for a suitable model description.

General linear models can be described symbolically by

$$y = Gu + He$$

where the measured output *y(t)* is the sum of a contribution from the measured input *u(t)* and a contribution from the noise He. The symbol G denotes the dynamic properties of the system, that is, how the output is formed from the input. For linear systems, G is the *transfer function* from input to output. The symbol H refers to the noise properties and it is called the *disturbance model*. H describes how the disturbances at the output are formed from some standardized noise source *e(t)*.

State-space models are common representations of dynamic models. They describe the same type of linear difference relationship between the inputs and the outputs as in the ARX model, but state-space models are rearranged so that only one delay is used in the expressions. To achieve this, additional variables, *state variables*, are introduced. State variables are not measured, but can be reconstructed from the measured input-output data. This is especially useful when there are several output signals, i.e., when *y(t)* is a vector. Chapter 3, "Tutorial," gives more details about this. For basic use of the toolbox, it is sufficient to know that the *order* of the state-space model relates to the number of delayed inputs and outputs used in the corresponding linear difference equation. The state-space representation looks like

$$x(t+1) = Ax(t) + Bu(t) + Ke(t)$$

$$y(t) = Cx(t) + Du(t) + e(t)$$

Here *x(t)* is the vector of state variables. The model order is the dimension of this vector. The matrix K determines the disturbance properties. Notice that if $K = 0$, then the noise source *e(t)* affects only the output, and no specific model of the noise properties is built. This case corresponds to $H = 1$ in the general linear model above, and is usually referred to as an *output-error model*. Notice that $D = 0$ means that there is no direct influence from *u(t)* on *y(t)*. Thus the effect of the input on the output all passes via *x(t)* and is delayed by at least one sample. The first value of the state variable vector *x(0)* reflects the initial conditions for the system at the beginning of the data record. When dealing with models in state-space form, you decide whether to estimate D, K, and *x(0),* or to set them to zero.

How to Interpret the Noise Source

In many cases of system identification, the effects of the noise on the output are insignificant compared to those of the input. With good signal-to-noise ratios

(SNR), it is less important to have an accurate disturbance model. Nevertheless, it is important to understand the role of the disturbances and the noise source *e(t)*, whether it appears in the ARX model or in the general descriptions given above.

When dealing with disturbances, it is important to

- Understand white noise
- Interpret the noise source
- Use the noise source when working with the model

How can you understand white noise? From a formal point of view, the noise source *e* is normally regarded as *white noise*. This means that it is entirely unpredictable. In other words, it is impossible to guess the value of *e(t)* no matter how accurately the past data up to time *t*-1 has been measured.

How can you interpret the noise source? The actual disturbance contribution to the output, *H e*, has real significance. It contains all the influences on the measured *y*, known and unknown, that are not contained in the input *u*. It explains the fact that even if an experiment is repeated with the same input, the output signal is typically somewhat different. However, the source of the noise, *e*, need not have any physical significance.In the airplane example mentioned earlier, the disturbance effects are wind gusts and turbulence. Describing these as arising from a white noise source via a transfer function *H* is just a convenient way of capturing their character.

How can you deal with the noise source when using the model? If the model is only used for simulation, i.e., the responses to various inputs are to be studied, then the disturbance model plays no immediate role. Because the noise source *e(t)* for new data is unknown, it is taken as zero in the simulations so as to study the effect of the input alone (a noise-free simulation). Making another simulation, with *e* being arbitrary white noise, will reveal how reliable the result of the simulation is but will not give a more accurate simulation result for the actual system's response. It is a different thing when the model is used for prediction: Predicting future outputs from inputs and previously measured outputs also means that future disturbance contributions have to be predicted. A known, or estimated, correlation structure for disturbances (which is really the disturbance model *H*) allows the prediction of future disturbances based on the previously measured values.

The need for and the usage of the noise model can be summarized as follows:

- It is, in most cases, required to obtain a better estimate for the dynamics, *G*.
- It indicates the reliability of noise-free simulations.
- It is required for reliable predictions and stochastic control design.

Terms to Characterize the Model Properties

The properties of an input-output relationship, such as the ARX model, follow from the numerical values of the coefficients and the number of delays used. This is, however, a fairly implicit way of talking about the model properties. In practice, the following terms are used:

Impulse Response

The impulse response of a dynamic model is the output signal that results when the input is an impulse; i.e., *u(t)* is zero for all values of *t* except *t=0*, where *u(0)*=1. It can be computed as in the equation following (ARX), by setting *t* equal to 0, T, 2T, ... , and by setting *y(-T)=y(-2T)*=0 and *u(0)*=1.

Step Response

The step response is the output signal that results from a step input; i.e., *u(t)* is 0 for negative values of *t* and 1 for positive values of *t*. The impulse and step responses together are called the model's *transient response*.

Frequency Response

The frequency response of a linear dynamic model describes how the model reacts to sinusoidal inputs. If the input *u(t)* is a sinusoid of a certain frequency, then the output *y(t)* is also a sinusoid of this frequency. The amplitude and the phase (relative to the input) will, however, be different. This frequency response is most often depicted by two plots: one that shows the amplitude change as a function of the sinusoid's frequency, and one that shows the phase shift as a function of frequency. This is known as a Bode plot.

Zeros and Poles

The zeros and the poles are equivalent ways of describing the coefficients of a linear difference equation, such as the ARX model. The poles relate to the output side and the zeros relate to the input side of this equation. The number of poles (or zeros) is equal to the number of sampling intervals between the most and least delayed output (or input). In the ARX model example in the beginning of this section, there are two poles and one zero.

The Basic Steps of System Identification

The system identification problem is to estimate a model of a system based on the observed input-output data. Several ways to describe a system and to estimate such descriptions exist. This section provides a brief account of the most important approaches.

The procedure to determine a model of a dynamic system from observed input-output data involves three basic ingredients:

- The input-output data
- A set of candidate models (the model structure)
- A criterion to select a particular model in the set, based on the information in the data (the identification method)

The typical identification process consists of stages where you iteratively select a model structure, compute the best model in the structure, and evaluate this model's properties. This cycle can be itemized, as follows:

1. Design an experiment and collect input-output data from the process to be identified.
2. Examine the data. Polish the data by removing trends and outliers, and select useful portions of the original data. You can also apply filters to the data to enhance important frequency ranges.
3. Select and define a model structure (a set of candidate system descriptions), within which a model is to be found.
4. Compute the best model in the model structure according to the input-output data and a given criterion for goodness of fit.
5. Examine the properties of the model obtained.
6. If the model is good enough, then stop; otherwise go back to step 3 to try another model structure. You can also try other estimation methods (step 4), or work further on the input-output data (steps 1 and 2).

The System Identification Toolbox offers several functions for each of these stages in the process.

For step 2, the System Identification Toolbox offers routines to plot the data, filter the data, and remove trends in the data, as well as to resample and reconstruct missing data.

For step 3, there are a variety of nonparametric models, the most common black-box input-output and state-space structures, as well as general tailor-made linear state-space models in discrete and continuous time.

For step 4, general prediction error (maximum likelihood) methods, as well as instrumental variable methods and subspace methods, are offered for parametric models, while basic correlation and spectral analysis methods are used for nonparametric model structures.

For examining the models in step 5, many functions are provided to compute and present frequency functions, poles, and zeros, as well as to simulate and predict with the model. There are also functions for transforming between continuous-time and discrete-time model descriptions, as well as to formats that are used in other toolboxes (such as the Control System Toolbox and the Signal Processing Toolbox).

A Startup Identification Procedure

There are no guaranteed strategies for creating good models in system identification. Given the number of possibilities, it is easy to get confused about what to do, what model structures to test, and so on. This section describes one strategy that often works well. The steps refer to functions within the GUI, but you can also go through them in command mode. For the basic commands, see Chapter 4, "Functions — By Category."

Step 1: Look at the Data

Plot the data. Look at it carefully and try to infer the dynamics. Can you see the effects in the outputs due to the changes in the input? Can you see nonlinear effects, such as different responses at different levels, or different responses to a step-up and a step-down? Are there portions of the data that appear to be messy or noninformative? Use these insights to select portions of the data for estimation and validation.

Do physical levels play a role in your model? If not, detrend the data by removing its mean. The models will then describe how changes in the input lead to changes in the output, but do not explain the actual levels of the signals. This is the normal situation.

The default situation with good data is that you detrend by removing the mean, then select the first half or so of the data record for estimation purposes, and use the remaining data for validation.

This is what happens when you select **Preprocess -> Quickstart** in the main **ident** window.

Step 2: Get a Feel for the Difficulties

Select **Estimate -> Quickstart** in the main **ident** window. This computes and displays the spectral analysis estimate, the correlation analysis estimate, a fourth-order ARX model with a delay estimated from the correlation analysis, and a default order state-space model computed by `n4sid`. This gives three plots.

Check the agreement between the following:

- Spectral analysis estimate and the frequency functions of the ARX and state-space models

- Correlation analysis estimate and the transient responses of the ARX and state-space models
- Measured validation data output and the simulated outputs of the ARX and state-space models

If the agreements are reasonable, then the problem is not so difficult and a relatively simple linear model will do the job. Proceed to step 4 and perform some fine-tuning of model orders and noise models, if necessary. Otherwise go to step 3.

Step 3: Examine the Difficulties

There can be several reasons why the comparisons in step 2 did not look good. This section discusses the most common ones and the approaches for handling them.

Model Is Unstable

The ARX or state-space model might turn out to be unstable, but could still be useful for control purposes. Change to a 5- or 10-step-ahead prediction instead of simulation in the **Model Output View**.

Feedback in Data

If there is feedback from the output to the input due to some regulator, then the spectral and correlation analysis estimates are not reliable. Discrepancies between these estimates and the ARX and state-space models can therefore be disregarded in this case. In the **Model Residuals View** of the parametric models, feedback in the data can appear as a correlation between residuals and input for negative lags.

Disturbance Model

If the state-space model is clearly better than the ARX model at reproducing the measured output, this is an indication that the disturbances have a substantial influence and it will be necessary to model them carefully.

Model Order

If a fourth-order model does not give a good **Model Output** plot, try using an eighth-order model. If the fit improves, it follows that higher order models are required but that linear models could be sufficient.

Additional Inputs

If the **Model Output** fit has not significantly improved by the approaches discussed thus far, think about the physics of the application. Are there more signals than have been, or could be, measured that might influence the output? If so, include these among the inputs and try a fourth-order ARX model from all the inputs again. (Note that the inputs need not be control signals; anything measurable, including disturbances, can be treated as inputs.)

Nonlinear Effects

If the fit between measured and model output is still bad, consider the physics of the application. Are there nonlinear effects in the system? In that case, form the nonlinearities from the measured data and add those transformed measurements as extra inputs. For example, if you realize that it is the electrical power that is the driving stimulus in a heating process, and temperature is the output, this could be as simple as forming the product of voltage and current measurements. What transformations you choose depends, of course, on the application. However, it does not take very much work to form a number of additional inputs by reasonable nonlinear transformations of the measured inputs, and just test whether including them improves the fit.

Still Problems?

If none of these tests leads to a model that reproduces the validation data reasonably well, the conclusion might be that a sufficiently good model cannot be produced from the data. There can be many reasons for this. It might be that the system has quite complicated nonlinearities that cannot be realized on physical grounds. In such cases, nonlinear, black-box models could be a solution. The most frequently used models of this character are the Artificial Neural Networks (ANN).

Another important reason for problems might be that the data does not contain sufficient information, for example, because of bad signal-to-noise ratios, large and nonstationary disturbances, and varying system properties.

Otherwise, use the insights from this step about suitable inputs and proceed to step 4.

Step 4: Fine-Tune Orders and Disturbance Structures

For real data there is no such thing as a correct model structure. However, different structures can result in different model quality. The only way to determine the model quality is to try different structures and then compare the resulting model properties. There are a few things to look for in these comparisons.

Fit Between Simulated and Measured Output

Keep the **Model Output View** open and look at the fit between the simulated output of the model and the measured output for the validation data. Formally, you could pick the model for which this number is the highest. In practice, it is better to be more pragmatic and also take into account the model complexity and whether the important features of the output response are captured.

Residual Analysis Test

If the model is a good model, the cross correlation function between residuals and input does not go significantly outside the confidence region. Otherwise there is something in the residuals that originates from the input and has not been properly taken care of by the model. A clear peak at lag k shows that the effect from input $u(t-k)$ on $y(t)$ is not correctly described. A rule of thumb is that a slowly varying cross-correlation function outside the confidence region is an indication of too few poles, while sharper peaks indicate too few zeros or wrong delays.

Pole-Zero Cancellations

If the pole-zero plot (including confidence intervals) indicates pole-zero cancellations in the dynamics, this suggests that lower order models can be used. In particular, if it turns out that the orders of ARX models have to be increased to get a good fit, but that pole-zero cancellations are indicated, then the extra poles are just introduced to describe the noise. In this case, try the ARMAX, OE, or BJ model structures with an A or F polynomial of an order equal to that of the number of noncanceled poles.

What Model Structures Should Be Tested?

It often takes just a few seconds to compute and evaluate a model in a certain structure, so you should have a generous attitude to performing the tests. However, experience shows that when the basic properties of the system's

behavior have been picked up, it is not much use to fine-tune orders just to improve the fit by a fraction of a percent.

Many ARX models: There is a cheap way to test many ARX structures simultaneously. Enter in the **Orders** text field many combinations of orders, using the colon (:) notation. You can also click the **Order Selection** button. When you select **Estimate**, models for all combinations (which could be hundreds) are computed and their (prediction error) fit to validation data is shown on a plot. By clicking in this plot, you insert the best models with any chosen number of parameters into the Model Board, and evaluate them as desired.

Many state-space models: A similar feature is also available for black-box state-space models, estimated using `n4sid`. When a good order has been found, try the PEM estimation method, which often improves the accuracy.

ARMAX, OE, and BJ models: Once you have a feel for suitable delays and dynamics orders, it is often useful to try out ARMAX, OE, and/or BJ with these orders and test some different orders for the disturbance transfer functions (C and D). The OE structure is especially suitable for poorly damped systems.

To study the problem further, you could consult the extensive literature available on order and structure selection.

Multivariable Systems

Systems with many input signals and/or many output signals are called *multivariable*. Such systems are often more challenging to model. In particular systems with several outputs could be difficult. A basic reason for the difficulties is that the couplings between several inputs and outputs lead to more complex models. The structures involved are richer, and more parameters will be required to obtain a good fit.

Available Models

The System Identification Toolbox as well as the GUI handle general, linear multivariable models. All models mentioned earlier are supported in the single-output, multiple-input case. For multiple outputs, ARX models and state-space models are covered. Multiple-output ARMAX and OE models are covered via state-space representations: ARMAX corresponds to estimating the K-matrix, while OE corresponds to fixing K to zero. (These are options in the GUI model order editor.)

Generally speaking, it is preferable to work with state-space models in the multivariable case, because the model structure complexity is easier to deal with. It is essentially just a matter of choosing the model order.

Working with Subsets of the Input-Output Channels

In the process of identifying good models of a system, it is often useful to select subsets of the input and output channels. Partial models of the system's behavior will then be constructed. It might not, for example, be clear whether all measured inputs have a significant influence on the outputs. You can most easily test that by removing an input channel from the data, building a model for how the outputs depend on the remaining input channels, and checking whether there is a significant deterioration in the fit of the model output from the measured one. See also the discussion under Step 3 above.

Generally speaking, the fit gets better when more inputs are included and often gets worse when more outputs are included. To understand the latter fact, you should realize that a model that has to explain the behavior of several outputs has a tougher job than one that must just account for a single output. If you have difficulties obtaining good models for a multioutput system, it might be wise to model one output at a time, to find out which are the difficult ones to handle.

Models that are just to be used for simulations could very well be built up from single-output models, for one output at a time. However, models for prediction and control produce better results if constructed for all outputs simultaneously. This follows from the fact that knowing the set of all previous output channels gives a better basis for prediction than just knowing the past outputs in one channel. Also, for systems where the different outputs reflect similar dynamics, using several outputs simultaneously will help estimating the dynamics.

Some Practical Advice

Both the GUI and command-line operation will do useful bookkeeping for you, handling different channels. You could follow these steps:

1. Import data and create a data set with all input and output channels of interest. Preprocess this set in terms of detrending, etc., and then select a validation data set with all channels.

2. Then select a working data set with all channels, and estimate state-space models of different orders, using `n4sid` for these data. Examine the resulting model primarily using the **Model Output** view.

3. If it is difficult to get a good fit in all output channels or you would like to investigate how important the different input channels are, construct new data sets using subsets of the original input/output channels. Use the menu item **Preprocess -> Select Channels** for this. Don't change the validation data. The GUI will keep track of the input and output channels. It does the right thing when evaluating the channel-restricted models using the validation data. It might also be appropriate to see if improvements in the fit are obtained for various model types, built for one output at a time.

- If you decide on a multioutput model, it is often easiest to use state-space models. Use `n4sid` as a primary tool and try `pem` when a good order has been found.

Reading More About System Identification

There is substantial literature on system identification. The following textbook deals with identification methods from a perspective like this toolbox's, and also describes methods for physical modeling:

- Ljung, L., and T. Glad, *Modeling of Dynamic Systems,* Prentice Hall, Englewood Cliffs, N.J., 1994.

For more details about the algorithms and theories of identification,

- Ljung, L., *System Identification – Theory for the User*, Prentice Hall, Upper Saddle River, N.J., 2nd edition, 1999.
- Söderström, T., and P. Stoica, *System Identification*, Prentice Hall International, London, 1989.

For a treatment on frequency domain data in particular,

Pintelon, R., and J. Schoukens, System Identification. A Frequency Domain Approach, IEEE Press, New York, 2001.

For more about system and signals,

- Oppenheim, J., and A.S. Willsky, *Signals and Systems,* Prentice Hall, Englewood Cliffs, N.J., 1985.

The following textbook deals with the underlying numerical techniques for parameter estimation:

- Dennis, J.E., Jr., and R.B. Schnabel, *Numerical Methods for Unconstrained Optimization and Nonlinear Equations*, Prentice Hall, Englewood Cliffs, N.J., 1983.

2

The Graphical User Interface

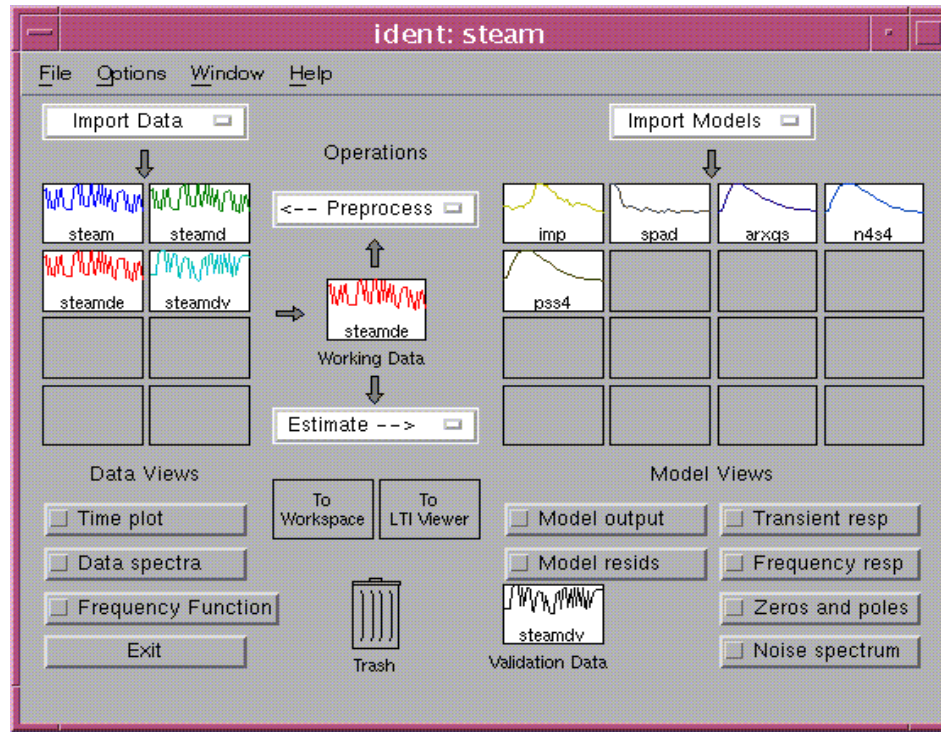
The Big Picture (p. 2-2)	A quick overview of the Ident GUI
Handling Data (p. 2-7)	Importing, preprocessing, and viewing data
Estimating Models (p. 2-16)	A discussion of direct and parametric identification methods
Examining Models (p. 2-31)	Examining, comparing, and validating identified models using frequency and transient responses, poles and zeros, and model outputs and residuals
Additional GUI Topics (p. 2-39)	Additional topics about the Ident GUI, including troubleshooting and customizing plots, reconfiguring the default layout, and limitations of the Ident GUI

The Big Picture

The System Identification Toolbox provides a graphical user interface (GUI). The GUI covers most of the toolbox functions and provides easy access to all variables that are created during a session. Start the session by typing

```
ident
```

in the MATLAB® Command Window.



The Main ident Window

The Model and Data Boards

System identification is about data and models and creating models from data. The main information and communication window, **ident**, is therefore dominated by two tables:

- A table of available data sets, each represented by an icon
- A table of created models, each represented by an icon

These tables are referred to as the *model board* and the *data board* in this chapter. You enter data sets in the data board by

- Opening earlier saved sessions
- Importing them from the MATLAB workspace
- Creating them by detrending, filtering, transforming, and selecting subsets of another data set in the data board

Imports are handled under the **Import data** menu while creation of new data sets is handled under the **Preprocess** menu. “Handling Data” on page 2-7 deals with this in more detail. The GUI supports three kinds of data objects for estimation. Both objects and vector- or matrix-valued signals can be imported:

- Time-domain input/output signals (in the `iddata` object format). These are marked by a white background color.
- Frequency-domain input/output signals (in the `iddata` object format). These are marked by a light green background color.
- Frequency functions (in the `idfrd` object format). These are estimates of the system’s frequency function (frequency response), obtained either by special data acquisition equipment (frequency analyzers) or as estimates from measured input-output data. These data sets are marked by a light brown background color.

You enter the models into the summary board by

- Opening earlier saved sessions
- Importing them from the MATLAB workspace
- Estimating them from data

Imports are handled under the **Import models** menu, while all the different estimation schemes are under the **Estimate** menu. More about this is in “Estimating Models” on page 2-16.

You can rearrange the data and model boards by dragging and dropping. More boards open automatically when necessary or when asked for (under the **Options** menu).

Working Data

All data sets and models are created from the working data set. This is the data in the center of the **ident** window. To change the working data set, drag and drop any data set from the data board on the working data icon.

Views

Below the data and model boards are buttons for various views. These control what aspects of the data sets and models you would like to examine, and are described in more detail in “Handling Data” on page 2-7 and in “Examining Models” on page 2-31. To select a data set or a model so that its properties are displayed, click its icon. A selected object is marked by a thicker line in the icon. To clear it, click again. You can examine an arbitrary number of data/model objects simultaneously. To obtain more information about an object, double-click (or right-click or **Ctrl**+click) its icon.

Validation Data

The two model views **Model Output** and **Model Residuals** illustrate model properties when applied to the validation data set. This is the set indicated in the box below these two views. To change the validation data, drag and drop any data set from the data board on the validation data icon.

It is good and common practice in identification to evaluate an estimated model’s properties using a fresh data set, that is, one that was not used for the estimation. It is thus good advice to let the validation data be different from the working data, but they should of course be compatible.

System Identification Workflow

Start by importing data (under the **Data** menu); examine the data set using the **Data Views**. You probably remove the means from the data and select subsets of data for estimation and validation purposes, using the items in the **Preprocess** menu. You then continue to estimate models, using the possibilities under the **Estimate** menu, perhaps first doing a quick start. You examine the obtained models with respect to your favorite aspects using various **Model Views**. The basic idea is that any selected view shows the properties of all selected models at any time. This function is live, so you can check models and views in and out at will. You select/deselect a model by clicking its icon.

Inspired by the information you gain from the plots, you continue to try out different model structures (model orders) until you find a model you are satisfied with.

Session Management

Diary: It is easy to forget what you have been doing. By double-clicking a data/model icon, you get a complete diary of how this object was created, along with other key information. At this point you can also add comments and change the name of the object and its color.

Layout: To have a good overview of the created models and data sets, it is good practice to try rearranging the icons by dragging and dropping. In this way models corresponding to a particular data set can be grouped together, etc. You can also open new boards (**Options -> Extra model/data boards**) to further rearrange the icons. These can be dragged across the screen between different windows. The extra boards are also equipped with notepads for your comments.

Sessions: The model and data boards with all models and data sets, together with their diaries, can be saved (under the **File** menu) at any point, and reloaded later. This is the counterpart of save/load workspace in the command-driven MATLAB. The four most recent sessions are listed under **File**.

Cleanliness: The boards will hold an arbitrary number of models and data sets (by creating clones of the board when necessary). However, you should clear (delete) models and data sets that are no longer of interest. Do that by dragging the object to the trash can icon. (Double-clicking the trash can opens it. You can retrieve its contents.) Empty the trash can if you run into memory problems.

Warnings: Several messages from the underlying computations can show up in warning dialog boxes. You can turn off these warnings using an item in the **Options** menu.

Window Culture: Dialog box and plot windows are best managed by the GUI's close function (item under the **File** menu). Alternatively, select **Close** or select/clear the corresponding **View** box. It is generally not recommended to minimize the windows, but to use the GUI's handling and window management system instead.

Workspace Variables

The models and data sets created within the GUI are normally not available in the MATLAB workspace. Indeed, the variables used during the system identification sessions do not automatically end up in the workspace. You can, however, export the variables to the workspace at any time, by dragging and dropping the data or model icons to the **To Workspace** icon. The corresponding workspace variables have the same name as the data or model you export. You can work with the variables in the workspace by using any MATLAB commands, and then you can import the modified versions back into the GUI. Note that models and data are exported as the toolbox objects `idmodel`, `idfrd`, and `iddata`. To learn how to extract information and work with these objects, see "Data Representation" on page 3-19 and "Model Conversion" on page 4-15.

The GUI's names of data sets and models are suggested by default procedures. Normally, you can enter any other name of your choice at the time of creation of the variable. You can change the names (after double-clicking the icon) at any time. Unlike the workspace situation, two GUI objects can carry the same name (i.e., the same string in their icons).

Context-Sensitive Help

The main **ident** window and the plot windows contain help topics under the **Help** menu. In addition, every dialog box has a **Help** button that provides help on that specific GUI.

Handling Data

Data Representation

In the System Identification Toolbox, signals and observed data are represented as column vectors, for example:

$$u = \begin{bmatrix} u(1) \\ u(2) \\ \dots \\ \dots \\ u(N) \end{bmatrix}$$

The entry in row number k, i.e., $u(k)$, will then be the signal's value at sampling instant number k. It is generally assumed in the toolbox that data is sampled at equidistant sampling times, and the sampling interval T is supplied as a specific argument.

For frequency-domain data, $u(k)$ is interpreted as the Fourier transform of the input at frequency $w(k)$, where the frequency vector w is defined along with the input.

The input to a system is generally denoted by the letter u and the output by y. If the system has several input channels, the input data is represented by a matrix, where the columns are the input signals in the different channels.

$$u = \begin{bmatrix} u1 & u2 & \dots & um \end{bmatrix}$$

The same holds for systems with several output channels.

The observed input-output data record is represented in the System Identification Toolbox by the `iddata` object that is created from the input and output signals by

```
Data = iddata(y,u,Ts)
```

where `Ts` is the sampling time. For frequency-domain data, the object is defined as

```
Data = iddata(y,u,Ts,'Domain','Frequency','Freq',w)
```

where `w` is the vector of frequencies.

The `iddata` object can also be created from the input and output signals when the data is inserted into the GUI.

Another data representation that can be used for model estimation is frequency responses (frequency functions). This consists of the frequency response from input to output `G(w)` (which the transfer function evaluated on the unit circle or the imaginary axis). The frequency function `G(w)` is a complex number, whose absolute value describes how a sinusoid of frequency `w` is amplified by the system, and whose argument (phase) describes how the same signal is shifted (phase-lagged) by the system. Frequency response data is contained in the `idfrd` object:

```
Datfr = idfrd(G,w,Ts)
```

`idfrd` objects can also be created from the basic signals when they are imported into the GUI.

Getting Data into the GUI

To bring data into the GUI, select the **Import Data** menu in the main GUI window. This gives you three choices:

- Time-domain data
- Frequency-domain data (also covers frequency response data)
- Data object

Depending on what you choose, slightly different dialog box windows open.

For input/output data, the information about a data set that should be supplied to the GUI is as follows:

- Input and output signals
- Name you give to the data set
- Sampling interval `Ts` (`Ts = 0` denotes time continuous data, which then must be frequency domain.)

For frequency-domain data, you also have to supply the

- Frequency vector

In addition to this mandatory information, you can add further properties that will help in the bookkeeping:

- Starting time for the sampling (For frequency-domain data, enter instead the frequency unit, Hz or rad/s.)
- Input and output channel names
- Input and output channel units
- Periodicity and intersample behavior of the input
- Data notes: These are notes for your own information and bookkeeping that will follow the data and all models created from it.

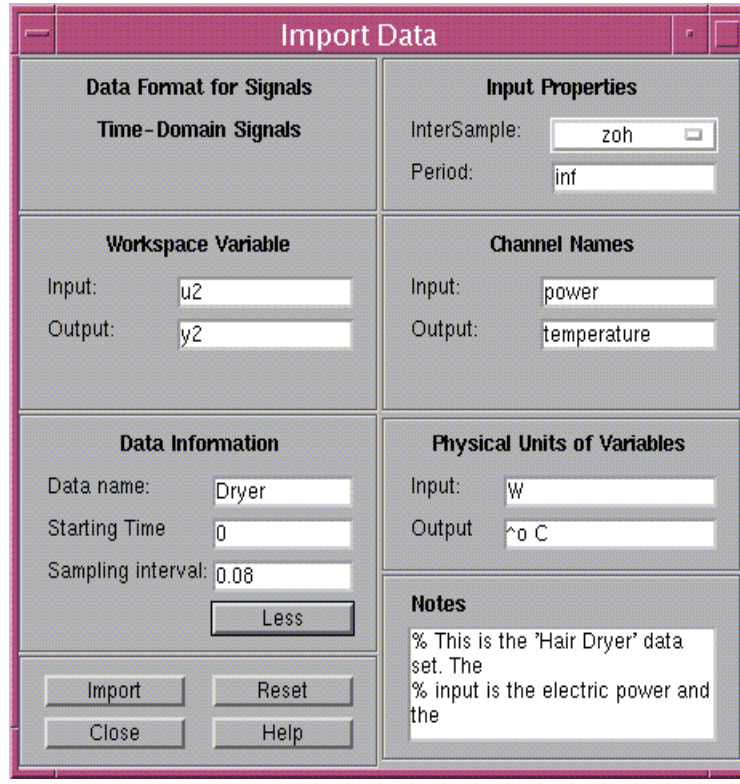
Note that the sampling interval and the input intersample properties are relevant also for frequency-domain data, because they determine how to interpret the information in the data.

In the case of frequency-response data, you have to enter

- The response function, either as a vector of complex values or as amplitude and phase
- The corresponding vector of frequencies
- The underlying sampling interval `Ts`. Use `Ts = 0` if the response corresponds to a continuous time system.

If the system has *nu* inputs and *ny* outputs, and the response is given at *nf* frequencies, the response function is an *ny*-by-*nu*-by-*nf* 3-D array. In this case also, you can supply bookkeeping information as above.

As you select the **Import Data** menu and choose the relevant item, a dialog box opens where you can enter the information.



Dialog box for Importing Data into the GUI

For the time-domain data case, the fields are as follows:

Input and **Output**: Enter the variable names of the input and output respectively. These should be variables in your MATLAB workspace, so you might have to load some disk files first.

Actually, you can enter any MATLAB expressions in these fields, and they are evaluated to compute the input and the output before the data is imported into the GUI.

Data name: Enter the name of the data set to be used by the GUI. You can change this name later on.

Starting time and **Sampling interval**: Fill these out for correct time and frequency scales in the plots.

Clicking **More** expands the dialog box and provides additional options:

Channel names: Enter strings for the different input and output channel names. Separate the strings by commas. The number of names must be equal to the number of channels. If these entries are not filled out, default names, for example, `y1`, `y2`, `...`, `u1`, `u2`, `...`, are used.

Channel units: Enter, in analogous format, the units in which the measurements are made. These will follow all models built from data, but are used only for plot information.

Period: If the input is periodic, enter the period length. `Inf` means a nonperiodic input, which is the default.

Intersample: Choose the intersample behavior of the input as ZOH (zero-order hold, i.e., the input signal is piecewise constant between the samples) or FOH (first-order hold, i.e., the input signal is piecewise linear between the samples) or BL (band-limited, i.e., the continuous-time input signal has no power above the Nyquist frequency). ZOH is the default.

The box at the bottom is for **Notes,** where you can enter any text you want to accompany the data for bookkeeping purposes.

Finally, select **Import** to insert the data into the GUI. When no more data sets are to be inserted, select **Close** to close the dialog box. Selecting **Reset** will empty all the fields of the box.

The procedure just described creates an `iddata` object, with all its properties (or correspondingly an `idfrd` object, in the frequency-response data case). If you already have an `iddata` or `idfrd` object available in the workspace, you can import that directly by selecting the item **Data Object** in the **Import Data** menu.

Taking a Look at the Data

The first thing to do after inserting the data set into the data board is to examine it. Selecting **Data View>Time plot** shows a plot of the input and output signals for the data sets that are selected. You select/clear the data sets by clicking them. For multivariable data, you can choose the different combinations of input and output signals under the **Channel** menu in the plot

window. Using the `zoom` function (drawing rectangles with the left mouse button down), you can examine different portions of the data in more detail.

To examine the frequency contents of the data, select **Data spectra** under **Data Views** in the ident window. The function is analogous to **Time plot**, but the signals' spectra are shown instead. By default the periodograms of the data are shown, i.e., the absolute squares of the Fourier transforms of the data. You can change the plot to any chosen frequency range and a number of different ways of estimating spectra, using the **Options** menu in the spectra window.

The purpose of examining the data in these ways is to find out if there are portions of the data that are not suitable for identification, if the information contents of the data are suitable in the interesting frequency regions, and if the data has to be preprocessed in some way before it can be used for estimation.

Another way of examining data is **Frequency Function** under **Data Views** in the ident window. For a frequency response data set this shows the amplitude and phase of the frequency function. For time- or frequency-domain input/output data, this view shows the empirical transfer function estimate (`etfe`) based on the data.

Preprocessing Data

The **Preprocess** menu has a number of methods to modify and transform the data sets on the data board. The commands are applied to the currently selected working data. The actual menu of choices depends on this data set. Not all choices are applicable to all kinds of data sets.

Detrending

Detrending the data involves removing the mean values or linear trends from the signals (the means and the linear trends are computed and removed from each signal individually). You access this function under the **Preprocess** menu by selecting **Remove Means** or **Remove Trends**. More advanced detrending, such as removing piecewise linear trends or seasonal variations, cannot be accessed within the GUI. We generally recommend that you remove at least the mean values of the data before the estimation phase. There are, however, situations when it is not advisable to remove the sample means. It could be, for example, that the physical levels are built into the underlying model, or that integrations in the system must be handled with the right level of the input being integrated.

Selecting Data Ranges

It is often the case that the whole data record is not suitable for identification, because of various undesired features (missing or bad data, outbursts of disturbances, level changes, etc.), so that only portions of the data can be used. In any case, it is advisable to select one portion of the measured data for estimation purposes and another portion for validation purposes. Selecting **Preprocess > Select Range** opens a dialog box that facilitates the selection of different data portions. You can type in the ranges or mark them by drawing rectangles with the mouse button down.

For multivariable data it is often advantageous to start by working with just some of the input and output signals. Selecting **Preprocess > Select Channels** allows you to select subsets of the inputs and outputs. This is done in such a way that the input/output numbering and names remain consistent when you evaluate data and model properties, for models covering different subsets of the data.

Prefiltering

By filtering the input and output signals through a linear filter (the same filter for all signals) you can, for example, remove drift and high-frequency disturbances in the data, which could have a bad influence on the model. You do this by selecting **Preprocess > Filter** in the main ident window. The dialog box is similar to the one where you select data ranges in the time domain. You mark with a rectangle in the spectral plots the intended passband or stop band of the filter, you select a button to check whether the filtering has the desired effect, and then you insert the filtered data into the GUI's data board.

Prefiltering is a good way of removing high-frequency noise in the data, and is also a good alternative to detrending (by cutting out low frequencies from the passband). Depending on the intended model use, you can also make sure that the model concentrates on the important frequency ranges. For a model that will be used for control design, for example, the frequency band around the intended closed-loop bandwidth is of special importance.

If you intend to use the data to build models both of the system dynamics and the disturbance properties, we recommend that you do the filtering at the estimation phase. Select **Estimate > Parametric Models**, and then select the estimation **Focus** to be **Filter**. This opens the same filter dialog box as above. The prefiltering, however, applies only for estimating the dynamics from input to output. The disturbance model is determined from the original data.

Resampling

If the data turns out to be sampled too fast, it can be decimated; i.e., every kth value is picked, after proper prefiltering (antialias filtering). This is obtained using **Preprocess > Resample**.

You can also resample at a faster sampling rate by interpolation, using the same command, and giving a resampling factor of less than 1.

Transform Data

Preprocess > Transform Data opens a dialog box that allows you to transform between time- and frequency-domain input/output data and also to form frequency-response data sets from input/output data.

Quickstart

Preprocess > Quickstart performs the following sequence of actions: It opens the **Time plot Data view**, removes the means from the signals, and splits this detrended data into two halves. The first one is made working data and the second one becomes validation data. All three created data sets are inserted into the data board.

Multiexperiment Data

The System Identification Toolbox allows the handling of data sets that contain several different experiments. Both estimation and validation can be applied to such data sets. This is quite useful to deal with experiments that have been conducted at different occasions but describe the same system. It is also useful to be able to keep together pieces of data that have been obtained by cutting out informative pieces from a long data set. Multiexperiment data can be imported and used in the GUI like any `iddata` object. Selecting a specific part of a multiexperiment data set is done using **Preprocess > Select Experiment**. To merge several data sets in the data board (obtained, for example, by cutting out portions from other data sets) use **Preprocess > Merge Experiment**.

Checklist for Data Handling

- Insert data into the GUI's data board.
- Plot the data and examine it carefully.
- Typically detrend the data by removing mean values.

- Select portions of the data for estimation and for validation. Drag and drop these data sets to the corresponding boxes in the GUI.

Simulating Data

The GUI is intended primarily for working with real data sets, and does not itself provide functions for simulating synthetic data. That has to be done in command mode, and you can use your favorite procedure in Simulink, the Signal Processing Toolbox, or any other toolbox for simulation, and then insert the simulated data into the GUI as described above.

The System Identification Toolbox also has several commands for simulation. For example, see the reference pages for `idinput` and `sim` for details.

The following example shows how the ARMAX model

$$y(t) - 1.5y(t-1) + 0.7y(t-2) =$$
$$u(t-1) + 0.5u(t-2) + e(t) - e(t-1) + 0.2e(t-1)$$

is simulated with a random binary input `u` and Gaussian noise `e`.

```
% Create an ARMAX model
model1 = idpoly([1 -1.5 0.7],[0 1 0.5],[1 -1 0.2]);
u = idinput(400,'rbs',[0 0.3]);
y = sim(model1,u,'noise');
```

The input, `u`, and the output, `y`, can now be imported into the GUI as data, and the various estimation routines can be applied to them. If you also import the simulation model `model1` into the GUI, its properties can be compared to those of the different estimated models.

To simulate a continuous-time state-space model

$$\dot{x} = Ax + Bu + Ke$$
$$y = Cx + e$$

with the same input, and a sampling interval of 0.1 second, do the following in the System Identification Toolbox:

```
A = [-1 1;-0.5 0]; B = [1; 0.5]; C = [1 0]; D = 0; K = [0.5;0.5];
Model2 = idss(A,B,C,D,K,'Ts', 0) % Ts = 0 means continuous time
Data = iddata([],u);
Data.Ts = 0.1
y=sim(Model2,Data,'noise');
```

Estimating Models

The Basics

Estimating models from data is the central activity in the System Identification Toolbox. It is also the one that offers the most possibilities and thus is the most demanding one for the user.

All estimation routines are accessed from the **Estimate** menu in the **ident** window. The models are always estimated using the data set that is currently in the **Working Data** box.

You can distinguish between two different types of estimation methods:

- Direct estimation of the impulse or the frequency response of the system. These methods are often called nonparametric estimation methods, and do not impose any structure assumptions about the system other than that it is linear.
- Parametric methods. A specific model structure is assumed, and the parameters in this structure are estimated using data. This opens up a large variety of possibilities, corresponding to the different ways of describing the system. The most important model structures include the state-space description, as well as several variants of difference equation descriptions.

Direct Estimation of the Impulse Response

A linear system can be described by the impulse response g_k, with the property that

$$y(t) = \sum_{k=1}^{\infty} g_k u(t-k)$$

The name derives from the fact that if the input *u(t)* is an impulse, i.e., *u(t)*=1 when *t*=0 and 0 when *t*>0, then the output *y(t)* will be $y(t) = g_t$. For a multivariable system, the impulse response g_k will be an *ny*-by-*nu* matrix

where *ny* is the number of outputs and *nu* is the number of inputs. Its *i-j* element thus describes the behavior of the *i*th output after an impulse in the *j*th input.

Choosing menu item **Estimate -> Correlation Model** opens a dialog box that lets you directly estimate the impulse response coefficients from the input/output data using so called *correlation analysis*. The actual method is described under the command `impulse` in Chapter 4, "Functions — By Category." For a quick action, you can also just type the letter `c` in the **ident** window. This is the *hot key* for correlation analysis.

The resulting impulse response estimate is placed in the model board, under the default name `imp`. (You can change the name by double-clicking the model icon and then typing in the desired name in the dialog box that opens.)

The best way to examine the result is to select **Transient Response** under **Model Views**. This gives a graph of the estimated response. This view offers a choice between displaying the impulse or the step response. For a multivariable system, the different channels, i.e., the responses from a certain input to a certain output, are selected under the **Channel** menu.

The number of lags for which the impulse response is estimated, i.e., the length of the estimated response, is determined as one of the options in the **Transient Response** view.

Direct Estimation of the Frequency Response

The frequency response of a linear system is the Fourier transform of its impulse response. This description of the system gives considerable engineering insight into its properties. The relation between input and output is often written

$$y(t)=G(z)u(t)+v(t)$$

where G is the transfer function and v is the additive disturbance. The function

$$G(e^{i\omega T})$$

as a function of (angular) frequency ω is then the frequency response or frequency function. T is the sampling interval. If you need more details on the different interpretations of the frequency response, see "The System Identification Problem" on page 3-9 or any textbook on linear systems.

You can estimate the system's frequency response directly using *spectral analysis* by selecting **Estimate > Spectral Model** and then clicking the **Estimate** button in the dialog box that opens. The result is placed on the model board under the default name `spd`. The best way to examine it is to plot it using **Frequency Response** under **Model Views**. This view offers a number of different options on how to graph the curves. You can also select the frequencies for which to estimate the response by specifying the number of frequencies and the spacing (linear or logarithmic) in the **Spectral Model** dialog box. The spectral analysis command also estimates the spectrum of the additive disturbance *v(t)* in the system description. You can examine this estimated disturbance spectrum using **Model Views > Noise Spectrum**.

The spectral analysis estimate is stored as an `idfrd` object. If you need to work further with the estimates, you can export the model to the MATLAB workspace and retrieve the responses directly from this object or by using the `nyquist` or `bode` command. See `idfrd`, `bode`, and `nyquist` in Chapter 4, "Functions — By Category," for more information. (Export a model by dragging and dropping it over the **To Workspace** icon.)

A few options that affect the spectral analysis estimate can be set in the dialog box. The most important choice is the frequency resolution. This is a number, M (the size of the lag window), that affects the frequency resolution of the estimates. Essentially, the frequency resolution is about $2\pi/M$ radians/(sampling interval). The choice of M is a tradeoff between frequency resolution and variance (fluctuations). A large value of M gives good resolution but fluctuating and less reliable estimates. The default choice of M is good for systems that do not have very sharp resonances and might have to be adjusted for more resonant systems.

The options also offer a choice between the Blackman-Tukey windowing method `spa` (which is the default); a variant with frequency dependent resolution, `spafdr`; and a method based on smoothing direct Fourier transforms, `etfe`. `etfe` has an advantage for highly resonant systems in that it is more efficient for large values of M. It however has the drawbacks that it requires linearly spaced frequency values, does not estimate the disturbance spectrum, and does not provide confidence intervals. The actual methods are described in more detail in Chapter 4, "Functions — By Category," under `spa`, `spafdr`, and `etfe`. To obtain the spectral analysis model for the current settings of the options, you can just type the hot key `s` in the **ident** window.

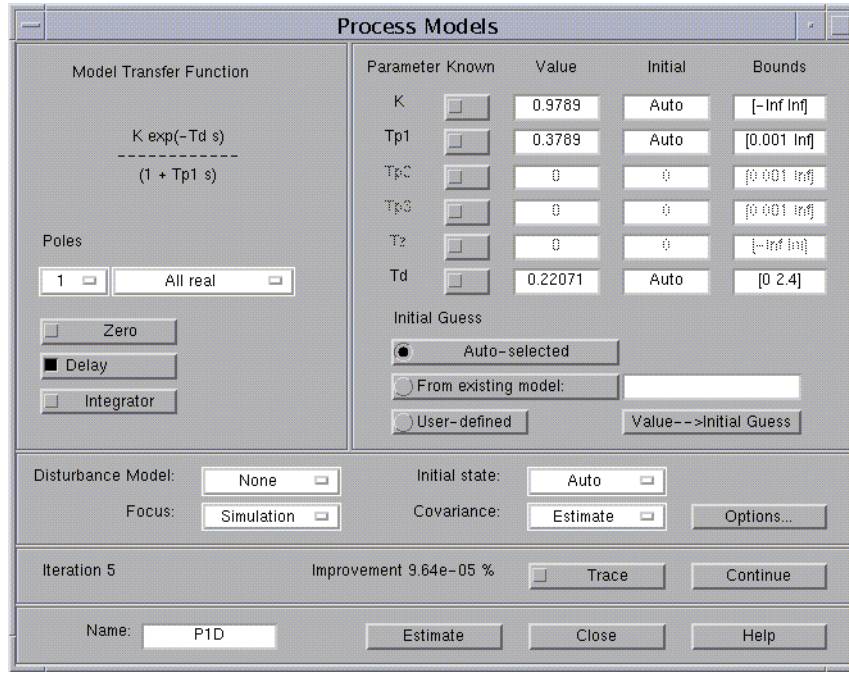
Estimation of Simple Process Model

The System Identification Toolbox allows you to estimate simple continuous-time process models characterizing the static gain, dominating time constants, and possible time delays (dead time). They are variants of the transfer function model structure

$$G(s) = \frac{K}{1 + sT_{p1}} e^{-sT_d}$$

where K is the static gain, T_{p1} is the time constant, and T_d is the delay.

To estimate models of this kind, select **Estimate > Process Models** in the **ident** window. This opens a dialog box as shown below.



Dialog Box for Estimating Process Models

In this dialog box you enter how many time constants (poles) to estimate and whether to include a time-delay term and an extra zero in the numerator of the transfer function. You can also enforce an integration for self-regulating processes. Moreover, there is a choice to force all time constants to be real or to allow underdamped modes (complex poles).

The dialog box can handle an arbitrary number of inputs, but only one output signal.

Some Further Estimation Options

The dialog box also has four menus that offer further options:

Disturbance Model allows you to include a first- or second-order model for the additive disturbances to the output.

Focus allows you to choose between a frequency weighting that concentrates on the model's prediction or simulation performance. Another alternative is prefiltering, which was described in "Prefiltering" on page 2-13.

The **InitialState** menu has options to estimate the initial state or to fix it to zero. The value **Auto** makes an automatic choice among these options.

The **Covariance** menu allows the choice between **Estimate** and **None**. Normally, the covariance of the model is estimated, so that various uncertainty measures can be displayed in the plots.

Initial Parameter Values and Parameter Bounds

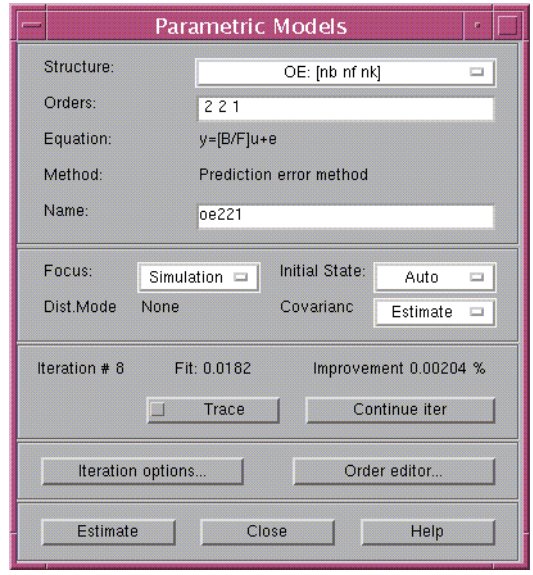
If no prior knowledge is available about the parameters, a startup routine is invoked to come up with initial parameter estimates. These are further iterated upon to give the best possible model fit to the data. The text AUTO is used to indicate that no initial guess is provided and an automatic process is invoked to estimate the initial values. If no qualified guess is available, this is usually a better alternative than entering an ad hoc value. However, if the estimation process gives parameter values that seem unreasonable, it might be worthwhile to try out various initial guesses and upper and/or lower limits of the parameters. Note that if you estimate a time delay, you must always provide an upper limit for the delay in order to secure efficient algorithms. The default value of this upper bound is 30 sampling intervals.

Iteration Information

The dialog box also provides information about the progress of the iterative optimization of the fit between the data and the model. The iteration number, the current fit information, and the improvement in fit (in percent) compared to the previous iteration are shown. You can also abort the iterations and save the current model, after the current iteration is finished. You access parameters that affect the minimization process by clicking **Options**.

Estimation of Parametric Models

The System Identification Toolbox supports a wide range of model structures for linear systems. Except for process models, they are all accessed by **Estimate > Parametric Models** in the **ident** window. This opens the **Parametric Models** dialog box, which contains the basic dialog box for all parametric estimation, as shown below.



Dialog Box for Estimating Parametric Models

The basic function of this box is as follows:

As you select **Estimate**, a model is estimated from the working data. The structure of this model is defined by the **Structure** menu together with the **Orders** edit box. It is given a name, which is written in the **Name** edit box.

The GUI will always suggest a default model name in the **Name** box, but you can change it to any string before clicking **Estimate**. (If you intend to export the model later, avoid spaces in the name.)

The interpretation of the model structure information (typically integers) in the **Orders** box depends on the structure selected in the menu. This covers, typically, six choices:

- ARX models
- ARMAX model
- Output-error (OE) models
- Box-Jenkins (BJ) models
- State-space models
- Model structure defined by initial model (user-defined structures)

You can fill out the **Orders** box yourself at any time, but for assistance you can select **Order Editor**. This opens another dialog box, depending on the chosen structure, in which you can enter the desired model order and structure information in a simpler fashion.

You can also enter the name of a MATLAB workspace variable in the **Orders** edit box. This variable should have a value that is consistent with the necessary orders for the chosen structure.

Note For the state-space structure and the ARX structure, you can enter several orders and combinations of orders. Then all corresponding models are compared and displayed in a special dialog box for you to select suitable ones. This could be a useful tool to select good model orders. This option is described in more detail later in this section. When it is available, an **Order selection** button is visible.

Estimation Method

A common method of estimating the parameters is the *prediction error approach*, where the parameters of the model are chosen so that the difference

between the model's predicted output and the measured output is minimized. This method is available for all model structures. Except for the ARX case, the estimation involves an iterative numerical search for the best fit.

Some information from this search is given online in the dialog box. By clicking **Iteration options**, you get access to a number of options that govern the search process. (See "Algorithm Properties" on page 4-15.)

For some model structures (the ARX model, and black-box state-space models) methods based on correlation are also available — Instrumental Variable (IV) and Subspace (N4SID) methods. The choice of methods is made in the **Parametric Models** dialog box.

The dialog box also has three menus that offer further options. **Focus** allows you to choose between a frequency weighting that concentrates on the model's prediction or simulation performance. Another alternative is prefiltering, which was described in "Prefiltering" on page 2-13. The **InitialState** menu has options to estimate the initial state or to fix it to zero. The value `Auto` makes an automatic choice among these options. Finally, the **Covariance** menu allows the choice between `Estimate` and `None`. Normally the covariance of the model is estimated, so that various uncertainty measures can be displayed in the plots. However, for high-order state-space models estimated by N4SID, or large multivariable ARX models, the computation of the covariance matrix can take quite a long time. Choosing **Covariance:** `None` greatly reduces the computation time.

Resulting Models

The estimated model is inserted into the GUI's model board. You can then examine its various properties and compare them with other models' properties using the **Model Views** plots. More about that is in "Examining Models" on page 2-31.

To take a look at the model itself, double-click the model's icon (or right-click or **Ctrl**+click). The **Data/Model Info** window that opens gives you information about how the model was estimated. You can also select the **Present** button, which lists the model and its parameters with estimated standard deviations in the MATLAB Command Window.

If you need to work further with the model, you can export it by dragging and dropping it over the **To Workspace** icon, and then apply any MATLAB and toolbox commands to it. (See, in particular, the commands `ssdata`, `tfdata`, `d2c`, and `get` in Chapter 4, "Functions — By Category.")

Which Structure and Method to Use

There is no simple way to find the best model structure; in fact, for real data, there is no such thing as a *best* structure. Some routes to find good and acceptable models are described in "A Startup Identification Procedure" on page 1-15. It is best to be generous at this point. It often takes just a few seconds to estimate a model, and, using the different validation tools described in the next section, you can quickly find out if the new model is any better than the ones you had before. There is often a significant amount of work behind the data collection, and spending a few extra minutes trying out several different structures is usually worthwhile.

ARX Models

Structure

The most used model structure is the simple linear difference equation

$$y(t) + a_1 y(t-1) + \ldots + a_{na} y(t-na) =$$
$$b_1 u(t-nk) + \ldots + b_{nb} u(t-nk-nb+1)$$

which relates the current output *y(t)* to a finite number of past outputs *y(t-k)* and inputs *u(t-k)*.

The structure is thus entirely defined by the three integers `na`, `nb`, and `nk`. `na` is equal to the number of poles and `nb 1` is the number of zeros, while `nk` is the pure time delay (the dead time) in the system. For a system under sampled-data control, typically `nk` is equal to 1 if there is no dead time.

For multiinput systems, `nb` and `nk` are row vectors, where the `i`th element gives the order/delay associated with the `i`th input.

Entering the Order Parameters

The orders `na`, `nb`, and `nk` can either be directly entered into the **Orders** edit box in the **Parametric Models** window, or selected using the menus in the **Order Editor**.

Estimating Many Models Simultaneously

By entering any or all of the structure parameters as vectors, using the MATLAB colon notation, such as `na=1:10`, you define many different structures that correspond to all combinations of orders. When you select

Estimate, models corresponding to all these structures are computed. A special plot window then opens that shows the fit of these models to validation data. By clicking in this plot, you can enter any models of your choice into the model board.

Multiinput models: For multiinput models you can enter each of the input orders and delays as a vector. The number of models resulting from all combinations of orders and delays can, however, be very large. As an alternative, you can enter one vector (such as `nb=1:10`) for all inputs and one vector for all delays. Then only models that have the same orders and delays from all inputs are computed.

Estimation Methods

There are two methods to estimate the coefficients `a` and `b` in the ARX model structure:

Least Squares: Minimizes the sum of squares of the right side minus the left side of the expression above, with respect to a and b. Select `ARX` in the **Method** box.

Instrumental Variables: Determines `a` and `b` so that the error between the right and left sides becomes uncorrelated with certain linear combinations of the inputs. Select `IV` in the **Method** box.

The methods are described in more detail in the reference pages for `arx` and `iv4`.

Multioutput Models

For a multioutput ARX structure with `ny` outputs and `nu` inputs, the difference equation above is still valid. The only change is that the coefficients a are ny-by-ny matrices and the coefficients b are ny-by-nu matrices.

The orders `[NA NB NK]` define the model structure as follows:

`NA`: an ny-by-ny matrix whose i-jth entry is the order of the polynomial (in the delay operator) that relates the jth output to the ith output

`NB`: an ny-by-nu matrix whose i-jth entry is the order of the polynomial that relates the jth input to the ith output

`NK`: an ny-by-nu matrix whose i-jth entry is the delay from the jth input to the ith output

The **Order Editor** dialog box allows the following choices

```
NA = na*ones(ny,ny)
NB = nb*ones(ny,nu)
NK = nk*ones(ny,nu)
```

where `na`, `nb`, and `nk` are chosen by the menus.

For custom order choices, construct a matrix `[NA NB NK]` in the MATLAB Command Window and enter the name of this matrix in the **Orders** edit box in the **Parametric Models** window.

Note that the possibility to estimate many models simultaneously is not available for multioutput ARX models.

See "Defining Model Structures" on page 3-39 for more information on multioutput ARX models.

ARMAX, Output-Error (OE), and Box-Jenkins (BJ) Models

There are several elaborations of the basic ARX model, where different disturbance models are introduced. These include well-known model types, such as ARMAX, output-error (OE), and Box-Jenkins (BJ).

The General Structure

A general input-output linear model for a single-output system with input `u` and output `y` can be written

$$A(q)y(t) = \sum_{i=1}^{nu} [B_i(q)/F_i(q)]u_i(t - nk_i) + [C(q)/D(q)]e(t)$$

Here u_i denotes input #i, and A, B_i, C, D, and F_i, are polynomials in the shift operator (z or q). (Don't be intimidated by this: It is just a compact way of writing difference equations; see below.)

You define the general structure by giving the time delays `nk` and the orders of these polynomials (i.e., the number of poles and zeros of the dynamic model from u to y, as well as of the disturbance model from e to y).

Special Cases

Most often the choices are confined to one of the following special cases:ARX: $A(q)y(t) = B(q)u(t - nk) + e(t)$

ARMAX: $A(q)y(t) = B(q)u(t-nk) + C(q)e(t)$

OE: $y(t) = [B(q)/F(q)]u(t-nk) + e(t)$ (output-error)

BJ: $y(t) = [B(q)/F(q)]u(t-nk) + [C(q)/D(q)]e(t)$ (Box-Jenkins)

The shift operator polynomials are just compact ways of writing difference equations. For example, the ARMAX model in longhand would be

$$y(t) + a_1 y(t-1) + \ldots + a_{na} y(t-na) = b_1 u(t-nk) + \ldots +$$
$$b_{nb} u(t-nk-nb+1) + e(t) + c_1 e(t-1) + \ldots + c_{nc} e(t-nc)$$

Note that *A(q)* corresponds to poles that are common to the dynamic model and the disturbance model (useful if disturbances enter the system close to the input). Likewise $F_i(q)$ determines the poles that are unique for the dynamics from input # i, and *D(q)* the poles that are unique for the disturbances.

The reason for introducing all these model variants is to provide for flexibility in the disturbance description and to allow for common or different poles (dynamics) for the different inputs.

Entering the Model Structure

Use the **Structure** menu in the **Parametric Models** dialog box to choose among the ARX, ARMAX, output-error, and Box-Jenkins structures. Note that if the working data set has several outputs, only the first choice is available. For time series (data with no input signal) only AR and ARMA are available among these choices. These are the time-series counterparts of ARX and ARMAX.

You select the orders of the polynomials using the menus in the **Order Editor** dialog box, or by directly entering them in the **Orders** edit box in the **Parametric Models** window. When the order editor is open, the default orders, entered as you change the model structure, are based on previously used orders.

Estimation Method

You estimate the coefficients of the polynomials using a prediction error/maximum likelihood method, by minimizing the size of the error term e in the expression above. Several options govern the minimization procedure. You access these by selecting **Iteration Options** in the **Parametric Models** window.

The algorithms are further described in Chapter 4, "Functions — By Category," under `armax`, `Algorithm Properties`, `bj`, `oe`, and `pem`. See also "Parametric Model Estimation" on page 3-28 and "Defining Model Structures" on page 3-39.

Note These model structures are available only for the scalar output case. For multioutput models, the state-space structures offer the same flexibility. Also note that it is not possible to estimate many different structures simultaneously for input-output models.

State-Space Models

The Model Structure

The basic state-space model in innovations form can be written

$$x(t+1) = A\,x(t) + B\,u(t) + K\,e(t)$$

$$y(t) = C\,x(t) + D\,u(t) + e(t)$$

The System Identification Toolbox supports two kinds of parameterizations of state-space models: black-box, free parameterizations and parameterizations custom-made for the application. The latter are discussed in "User-Defined Model Structures" on page 2-30. First the black-box case is described.

Entering Black-Box State-Space Model Structures

The most important structure index is the model order, i.e., the dimension of the state vector x.

Use the menu in the **Order Editor** to choose the model order, or enter it directly into the **Orders** edit box in the **Parametric Models** window. You can further affect the chosen model structure:

- Fixing K to zero gives an output-error method; i.e., the difference between the model's simulated output and the measured one is minimized. Formally, this corresponds to an assumption that the output disturbance is white noise. This is done by the menu under **Disturbance Model.**
- The delays from the input can be chosen independently for each input. It will be a row vector `nk`, with *nu* entries. When the delay is larger than or equal to one, the D-matrix in the discrete-time model is fixed to zero. For physical

systems without a pure time delay that are driven by piecewise constant inputs, `nk` = 1 is a natural assumption. This is also the default. You can set the delays `nk` either in the order editor or directly in the **Orders** box as numbers in square brackets.

- The initial state X0 can either be estimated, set to zero, or backcast. This is handled by the **Initial State** menu.

Estimating Many Models Simultaneously

If you enter a vector for the model order, using the MATLAB colon notation (such as 1:10), all indicated orders are computed using a preliminary method. You can then enter models of different orders into the model board by clicking in a special graph that contains information about the models.

Estimation Methods

There are two basic methods for the estimation.

`pem`: Standard prediction error/maximum likelihood method, based on iterative minimization of a criterion. The iterations are started up at parameter values that are computed from `n4sid`. The parameterization of the matrices *A, B, C, D,* and *K* is free. The search for minimum is controlled by a number of options. These are accessed from the **Option** button in the **Iteration Control** window.

`n4sid`: Subspace-based method that does not use iterative search. The quality of the resulting estimates can significantly depend on options `N4Weight` and `N4Horizon`. You choose these options in the **Order Editor** dialog box. If you enter `N4Horizon` with several rows, the models corresponding to the horizons in each row are examined separately using the working data. The best model in terms of prediction (or simulation, if $K = 0$) performance is selected. A figure is shown that illustrates the fit as a function of the horizon. If you leave the `N4Horizon` box empty, a default choice is made.

Note When you use Order Selection, the default N4Horizons will be chosen according to the highest order you asked for. The chosen values will be displayed in the Order Editor. If you reestimate the model with the same order later, other default N4Horizons may be used, resulting in a slightly different model.

See the reference pages for `n4sid` and `pem` for more information.

User-Defined Model Structures

State-Space Structures

The System Identification Toolbox supports user-defined linear state-space models of arbitrary structure. Using the `idss` model structure, known and unknown parameters in the *A, B, C, D, K,* and *X0* matrices can be easily defined both for discrete- and continuous-time models. The `idgrey` object allows you to use a completely arbitrary grey box structure, defined by an M-file. The model object properties can be easily manipulated. See the reference pages for `idss` and `idgrey` and "Structured State-Space Models with Free Parameters: the idss Model" on page 3-48.

To use these structures in conjunction with the GUI, just define the appropriate structure in the MATLAB Command Window. Then use the **Structure** menu to select `By Initial Model`, enter the variable name of the structure in the **Initial Model** edit box in the **Parametric Models** window, and select **Estimate**.

Any Model Structure

Arbitrary model structures can be defined using the System Identification Toolbox model objects:

- `idpoly:` Creates input-output structures for single-output models
- `idss`: Creates linear state-space models with arbitrary free parameters
- `idgrey`: Creates completely arbitrary parameterizations of linear systems
- `idproc`: Creates simple process models
- `idarx`: Creates multivariable ARX structures

To work with any such defined or estimated model in the GUI, use the **Structure** menu to select `By Initial Model`, enter the variable name of the structure in the **Initial Model** edit box in the **Parametric Models** window, and select **Estimate**. Then the parameters of the model structure are adjusted to the chosen working data set. The method is a standard prediction error/maximum likelihood approach that iteratively searches for the minimum of a criterion. You access the options that govern this search by the **Iteration Options** button in the **Parametric Models** window.

The name of the initial model must be a variable either in the workspace or in the model board. In the latter case you can just drag and drop it over the **Orders/Initial model** box.

Examining Models

Estimating a model is just a first step. Now you must examine it, compare it with other models, and test it with new data sets. You do this primarily using the six **Model Views** functions at the bottom of the main **ident** window:

- Frequency response
- Transient response
- Poles and zeros
- Noise spectrum
- Model output
- Model residuals

In addition, you can double-click the model's icon to get **Text Information** about the model. Finally, you can export the model to the MATLAB workspace and use any commands for further analysis and model use.

Views and Models

If a certain **View** window is open (selected), then *all models* in the model summary board that are selected will be represented in the window. You can click in and out of the curves in the **View** window by selecting and clearing the models in an online fashion. You select and clear a model by clicking its icon. A selected model is marked with a thicker line in its icon.

On color screens, the curves are color coded along with the model icons in the model board. Before printing a plot it might be a good idea to differentiate the line styles (menu item under **Style**). This could also be helpful on black and white screens.

Note that models that are obtained by spectral analysis only can be represented as frequency response and noise spectra, and that models estimated by correlation analysis only can be represented as transient response.

About Plot Views

The six views all give similar plot windows, with several common features. They have a common menu bar, which covers some basic functions.

First of all, note that there is a zoom function in the plot window. By dragging with the left mouse button down, you can draw rectangles, which are enlarged when the mouse button is released. Double-clicking restores the original axis scales. For plots with two axes, the *x*-axes scales are locked to each other. A single left-click zooms in by a factor of two, while the middle button zooms out. The zoom function can be deactivated if desired. Just select **Zoom** under **Style**.

Second, pointing to any curve in the plot and Shift-clicking identifies the curve with the model name and current coordinates.

The common menu bar covers the following functions:

File

File allows you to copy the current figure to another standard MATLAB figure window. This might be useful, for example, when you intend to print a customized plot. Other **File** items cover printing the current plot and closing the plot window.

Options

Options covers actions for setting the axes scaling. This menu item also provides choices that are specific for the current plot window, such as a choice between step response or impulse response in the **Transient response** window.

An important option is the possibility of showing confidence intervals. Each estimated model property has some uncertainty. This uncertainty can be estimated from data. When you select **Show confidence intervals**, a confidence region around the nominal curve (model property) is marked (by dash-dotted lines). You can also set the level of confidence using this menu item.

Note Confidence intervals are supported for most models and properties, except models estimated using `etfe` and the k-step-ahead prediction property. For `n4sid`, the covariance properties are actually not fully known. The Cramer-Rao lower limit for the covariance matrix is therefore used instead.

Style

The **Style** menu gives access to various ways of affecting the plot. You can add gridlines, turn the zoom on and off, and change the line styles. The menu also covers a number of other options, like choice of units and scale for the axis.

Channel

For multivariate systems, you can choose which input-output channel to examine. The current choice is marked in the figure title.

Help

The **Help** menu has a number of items that explain the plot and its options.

Frequency Response and Disturbance Spectra

All linear models that are estimated can be written in the form

$$y(t)=G(z)u(t)+v(t)$$

where *G(z)* is the (discrete-time) transfer function of the system and *v(t)* is an additive disturbance. The frequency response or frequency function of the system is the complex-valued function $G(e^{i\omega T})$ viewed as a function of angular frequency ω.

This function is often graphed as a Bode diagram. That is, the logarithm of the amplitude (the absolute value) of $G(e^{i\omega T})$ and the phase (the argument) of $G(e^{i\omega T})$ are plotted against the logarithm of frequency ω in two separate plots. These plots are obtained by selecting **Frequency Response** under **Model Views** in the main **ident** window.

You can plot the estimated spectrum of the disturbance *v* as a power spectrum by choosing **Noise Spectrum** under **Model Views.**

If the data is a time series *y* (with no input *u*), then the spectrum of *y* is plotted under the **Noise Spectrum**, and no frequency functions are given.

Transient Response

You can get insight into a model's dynamic properties by looking at its step response or impulse response. This is the output of the model when the input

is a step or an impulse. These responses are plotted when you select **Transient Response** under **Model Views**.

It is informative to compare the transient response of a parametric model with the one that was estimated using correlation analysis. If there is good agreement between the two, you can be confident that some correct features have been picked up. It is useful to check the confidence intervals around the responses to see what "good agreement" could mean quantitatively.

Many models provide a description of the additive disturbance *v(t)*:

$$v(t)=H(z)e(t)$$

Here *H(z)* is a transfer function that describes how the disturbance *v(t)* can be thought of as generated by sending white noise *e(t)* through it. To display the properties of H, you can choose channels (in the **Channel** menu) that have noise components as inputs. The names of these channels are like `e@ynam`, for the noise component of e that directly affects the output channel with name `ynam`.

Poles and Zeros

The poles of a system are the roots of the denominator of the transfer function *G(z)*, while the zeros are the roots of the numerator. In particular the poles have a direct influence on the dynamic properties of the system.

You plot the poles and zeros of G (and H) by choosing **Poles and Zeros** under **Model Views**.

It is useful to turn on the confidence intervals in this case. They will reveal which poles and zeros could cancel each other (their confidence regions overlap). That is an indication that you could use a lower order dynamic model.

For multivariable systems, it is the poles and zeros of the individual input/output channels that are displayed. To obtain the so-called transmission zeros, you must export the model and then apply the command `tzero`. (This requires the Control System Toolbox.)

Compare Measured and Model Outputs

A good way of obtaining insight into the quality of a model is to simulate the model with the input from a fresh data set and compare the simulated output with the measured one. This gives a good feel for which properties of the system have been picked up by the model, and which have not.

You obtain this test by selecting **Model Output** under **Model Views**. Then the data set currently in the **Validation Data** box is used for the comparison. The fit is also displayed. This is computed as the percentage of the output variation that is reproduced by the model. So a model that has a fit of 0% gives the same mean square error as just setting the model output to the mean of the measured output.

If the model is unstable, or has integration or very slow time constants, the levels of the simulated and the measured output can drift apart, even for a model that is quite good (at least for control purposes). Then it is a good idea to evaluate the model's predicted output rather than the simulated one. With a *prediction horizon* of k, the k-step-ahead predicted output is obtained as follows:

> The predicted value *y(t)* is computed from all available inputs $u(s)(s \le t)$ (used according to the model) and all available outputs up to time t-k,$y(s)(s \le t-k)$. The simulation case, where no past outputs at all are used, thus formally corresponds to k=∞. To check whether the model has picked up interesting dynamic properties, it is wise to let the predicted time horizon (kT, T being the sampling interval) be larger than the important time constants.

Note that different models use the information in past output data in their predictors in different ways, depending on the disturbance model. For example, so-called output-error models (obtained by fixing K to zero for state-space models and setting `na=nc=nd=0` for polynomial models) do not use past outputs at all. The simulated and the predicted outputs, for any value of k, thus coincide.

The character of the comparison depends on the type of validation data. For frequency-domain input/output validation data, the amplitudes of the measured output signal are shown together with the models' simulated output frequency response (which is the product of the input frequency domain signal and the model frequency response). In this case, the predictions are not applicable. For frequency-response data the amplitude of the frequency response data is compared to the models' frequency responses. Note that even though just the amplitudes are shown in the plots, the figure of fit refers to the distance between the functions as complex variables.

Residual Analysis

In a model

$$y(t) = G(z)u(t) + H(z)e(t)$$

the noise source *e(t)* represents that part of the output that the model could not reproduce. It gives the left-overs, or the *residuals*. For a good model, the residuals should be independent of the input. Otherwise, there would be more in the output that originates from the input and that the model has not picked up.

To test this independence, compute the cross-correlation function between input and residuals by selecting **Model Residuals** under **Model Views**. It is wise to also display the confidence region for this function. For an ideal model the correlation function should lie entirely between the confidence lines for positive lags. If, for example, there is a peak outside the confidence region for lag k, this means that there is something in the output *y(t)* that originates from *u(t-k)* and that has not been properly described by the model. The test is carried out using the validation data. If these were not used to estimate the model, the test is quite tough. See also "Model Structure Selection and Validation" on page 3-70.

For a model also to give a correct description of the disturbance properties (i.e., the transfer function H), the residuals should be mutually independent. This test is also carried out by the view **Model Residuals**, by displaying the autocorrelation function of the residuals (excluding lag zero, for which this function by definition is 1). For an ideal model, the correlation function should be entirely inside the confidence region.

For frequency-domain validation data, the power spectrum of the residuals is shown, as well as the amplitude of the estimated transfer function from inputs to residuals.

Text Information

Double-clicking (right mouse button or **Ctrl**+click) the model icon opens a **Data/Model Info** dialog box containing some basic information about the model. It also has a diary of how the model was created, along with the notes that originally were associated with the estimation data set. At this point you can do a number of things.

Present

Clicking the **Present** button displays details of the model in the MATLAB Command Window. The model's parameters along with estimated standard deviations are displayed, as well as some other notes.

Modify

You can type any text you want anywhere in the **Diary and Notes** field of the dialog box. You can also change the name of the model by editing the text field with the model name. The color associated with the model in all plots can also be edited. Enter RGB values or a color name (such as `'y'`) in the corresponding box.

LTI Viewer

If you have the Control System Toolbox, you will see a **To LTI Viewer** icon in the main window. Dragging and dropping a model onto this icon opens the LTI Viewer. This viewer handles an arbitrary amount of models, but it requires all of them to have the same number of inputs and outputs. Note that the LTI viewer is not fully live when you click in and out of the models on the model board. Instead, the LTI viewer has its own interface for dealing with models and channels (right-click in the plot).

Further Analysis in the MATLAB Workspace

You can export any model or data object to the MATLAB workspace by dragging and dropping its icon over the **To Workspace** box in the **ident** window.

Once you have exported the model to the workspace, there are many commands by which you can further transform it, examine it, and convert it to other formats for use in other toolboxes. Some examples of such commands are as follows:

`d2c`	Transform to continuous time
`ss, idss, ssdata`	Convert to state-space representation
`tf, tfdata`	Convert to transfer function form
`zpk, zpkdata`	Convert to zeros and poles

Note that the commands `ss`, `tf`, and `zkp` transform the model to the Control System Toolbox's LTI models. Moreover, if you have that toolbox, many of its LTI commands can be applied directly to the model objects of the System Identification Toolbox. See "Connections Between the Control System Toolbox and the System Identification Toolbox" on page 3-96.

If you need to prepare specialized plots that are not covered by the **Views**, all the System Identification Toolbox commands for computing and extracting simulations, frequency functions, zeros and poles, etc., are also available. See Chapter 3, "Tutorial" and Chapter 4, "Functions — By Category."

Additional GUI Topics

This section discusses a number of different topics.

Mouse Buttons and Hot Keys

The GUI uses three mouse buttons. If you have fewer buttons on your mouse, the actions associated with the middle and right mouse buttons are obtained by **Shift**+click, **Alt**+click, or **Ctrl**+click, depending on the computer.

The Main ident Window

In the main **ident** window the mouse buttons are used to drag and drop, to select/clear models and data sets, and to double-click to get text information about the object. You can use the left mouse button for all of this. A certain speedup is obtained if you use the left button for dragging and dropping, the middle one (equivalent to Shift-click) for selecting models and data sets, and the right one (equivalent to Ctrl-click) for double-clicking. Actually, for the right mouse button, a single-click is sufficient. On a slow machine a double-click from the left button might not be recognized.

The **ident** window also has a number of *hot keys*. By pressing a keyboard letter when the **ident** window is the current window, you can quickly activate some functions. These are

- `s`: Computes the **Spectral Analysis Model** using the current option settings. (You can change options in the dialog box window that opens when you choose **Estimate** > **Spectral Model**.)
- `c`: Computes **Correlation Analysis Model** using the current option settings.
- `q`: Computes the models associated with **Quickstart**.
- `d`: Opens a dialog box for importing data objects.

Plot Windows

In the various plot windows the action of the mouse buttons depends on whether zoom is activated or not.

If zoom is active. The left and middle mouse buttons are associated with the zoom functions, as in MATLAB. The left button zooms in and the middle one zooms out. In addition, you can draw rectangles with the left button to define

the area to be zoomed. Double-clicking restores the original plot. The right mouse button is associated with special GUI actions that depend on the window. In the **View** plots, the right mouse button is used to identify the curves. Point and click a curve, and a box displays the name of the model/data set that the curve is associated with, and also the current coordinate values for the curve. In the **Model Selection** plots, the right mouse button is used to inspect and select the various models. In the **Prefilter** and **Data Range** plots, rectangles are drawn with this mouse button down to define the selected range.

If zoom is not active. The special GUI functions just mentioned are obtained by any mouse button.

The zoom is activated and deactivated using the **Style** menu**.** The default setting differs between the plots.

Note Don't activate the zoom from the command line! That will destroy the special GUI functions. (If you happen to do so anyway, quit the window and open it again.)

Troubleshooting in Plots

The function **Auto-range** under the **Options** menu sets automatic scales to the plots. It is also a good function to invoke when you think that you have lost control over the curves in the plot. (This might happen, for example, if you zoom in a portion of a plot and then change the data of the plot.)

If the view plots don't respond the way you expect them to, you can always quit the window and open it again. By *quit* here is meant using the underlying window system's own quitting mechanism, which is called different things in the different platforms. The normal way to close a window is to use the **Close** function under the **File** menu or to clear the corresponding check box.

Layout Questions and idprefs.mat

The GUI comes with a number of preset defaults. These include the window sizes and positions, the colors of the different models, and the default options in the different **View** windows.

You can change the window sizes and positions, as well as the options in the plot windows, in the standard way. If you want the GUI to start with your

current window layout and current plot options, select **Options > Save preferences** in the main **ident** window. This saves the information in a file `idprefs.mat`. This file also stores information about the four most recent sessions with **ident**. This allows the session **File** menu to be correctly initialized. The session information is automatically stored upon exit. The layout and preference information is only saved when you select the indicated option.

The file `idprefs.mat` is created the first time you close the GUI. It is stored in the same directory as your `startup.m` file by default. If this default does not work, you are prompted for a directory to store the file. You can ignore this, but then you cannot save session and preference information.

To change or select a directory for `idprefs.mat`, use the command `midprefs`.

To change model colors and default options to your own customized choices, make a copy of the M-file `idlayout.m` to your own directory (which should be *before* the basic `ident` directory in the `MATLABPATH`), and edit it according to its instructions.

Customized Plots

If you need to prepare hardcopies of your plots with specialized texts, titles, and so on, make a copy of the figure first, using **File > Copy Figure**. This produces a copy of the current figure in a standard MATLAB figure format.

For plots that are not covered by the **View** windows (e.g., Nyquist plots), you have to export the model to the MATLAB workspace and construct the plots there.

What You Cannot Do Using the GUI

The GUI enables you to examine the data, estimate models, and evaluate and compare models. However, you cannot do the following in the GUI:

- Generate (simulate) data sets
- Create models (by methods other than estimation)

- Manipulate and convert models
- Use recursive (online) estimation algorithms

To see what M-files are available in the toolbox for these functions, see "Toolbox Commands" on page 3-3, as well as "Simulation and Prediction" on page 4-5, "Model Structure Creation" on page 4-10, "Manipulating Model Structures" on page 4-13, "Model Conversion" on page 4-15, and "Recursive Parameter Estimation" on page 4-12.

Note that at any point you can export a data set or a model to the MATLAB workspace (by dragging and dropping its icon on the **To Workspace** icon). There you can modify and manipulate it any way you want and then import it back into **ident**. You can, for example, construct a continuous-time model from an estimated discrete-time one (using `d2c`), and then use the model views to compare the two.

3

Tutorial

Overview (p. 3-2)	Quick look at the contents of this tutorial
Toolbox Commands (p. 3-3)	Overview of the organization of system identification commands
An Introductory Example to Command Model (p. 3-5)	Worked out example that uses only the command line
The System Identification Problem (p. 3-9)	Discussion of the basic issues in system identification
Data Representation and Nonparametric Model Estimation (p. 3-19)	Various ways to represent data
Parametric Model Estimation (p. 3-28)	Estimating models using parametric methods
Defining Model Structures (p. 3-39)	Overview of available model structures
Examining Models (p. 3-57)	How to plot the responses and further examine estimated models
Model Structure Selection and Validation (p. 3-70)	Discussion of the process used to select model structure, and how to validate an identified model
Dealing with Data (p. 3-81)	Collection of topics about data preprocessing, such as detrending and filtering
Recursive Parameter Estimation (p. 3-86)	Overview of the algorithms used in recursive estimation
Miscellaneous Topics (p. 3-93)	Various topics, including time-series modeling and connections between the System Identification Toolbox and the Control System Toolbox

Overview

This chapter has three purposes:

- It provides an overview of system identification theory, the basic models and disturbance descriptions used, and the character of the basic algorithms. It also provides some practical advice for a number of issues that are essential for a successful application.
- It describes the commands and objects of the System Identification Toolbox, their syntax and use. If you primarily use the graphical user interface (GUI), you will not have to bother about these aspects.
- It describes the commands that are not reached from the GUI, that is, simulation, the recursive algorithms, and more advanced model structure definitions.

Toolbox Commands

It might be useful to recognize several *layers* of the System Identification Toolbox. Initially concentrate on the first layer of basic tools, which contains the commands from the System Identification Toolbox that any user must master. You can proceed to the next levels whenever an interest or the need from the applications warrants it. The layers are described in the following paragraphs:

Layer 0: Help Functions. `Help ident` gives an overview of available commands. `idhelp` gives access to a micromanual of command-line help, with several subhelps like `idhelp evaluate`, etc. There is also a command `advice` that can be applied to any data set and any model.

```
advice(data)
advice(model)
```

This gives text information on the screen about the quality of the data/model and some advice on how to proceed.

Layer 1: Basic Tools for Estimating Black-Box Models. The first layer contains the basic tools for estimating models from measured data. It is necessary to know the basics of the data representation and the simple commands to build and evaluate black-box models. The commands are

Data representation	`iddata`, `plot`
Nonparametric estimation of impulse and frequency response	`impulse`, `step`, `spa`
Estimating black-box models of state-space and input-output type	`pem`, `arx`
Evaluating models	`compare`, `resid`
Displaying model characteristics	`bode`, `nyquist`, `pzmap`, `step`, `view`
Looking at parametric model characteristics	By field referencing, like `Mod.A`, `Mod.dA`

The corresponding background is given in the next few sections of this tutorial.

Layer 2: Creating Models for Simulation and Transforming Models. To define models, to generate inputs, and to simulate models,

```
idarx, idpoly, idproc, idss, idinput, sim
```

To transform models to other representations,

```
arxdata, polydata, ssdata, tfdata, zpkdata
```

Layer 3: Model Structure Selection. The third layer of the toolbox contains some useful techniques to select orders and delays.

```
arxstruc, selstruc
```

Layer 4: Structured Models and Further Model Conversions. The fourth layer contains transformations between continuous and discrete time, and functions for estimating completely general model structures for linear systems. The commands are

```
c2d, d2c, idss, idgrey, pe, predict
ss, tf, zp, frd
```
(to be used with the Control System Toolbox)

The corresponding material is covered in "Defining Model Structures" on page 3-39 and in "Examining Models" on page 3-57.

Layer 5: Recursive Identification. Recursive (adaptive, online) methods of parameter estimation are covered by the commands

```
rarmax, rarx, rbj, roe, rpem, rplr
```

They are covered in "Recursive Parameter Estimation" on page 3-86.

See Chapter 4, "Functions — By Category" for a complete list of available functions.

An Introductory Example to Command Model

A demonstration M-file called `iddemo.m` provides several examples of typical sessions with the System Identification Toolbox. To start the demo, execute `iddemo` from inside MATLAB.

Before giving a formal treatment of the capabilities and possibilities of the toolbox, this example is designed to get you started with the software quickly. This example is essentially the same as demo #2 in `iddemo`. You might want to invoke MATLAB at this time, execute the demo, and follow along.

Example Details

Data has been collected from a laboratory scale process. (Feedback's Process Trainer PT326; see page 526 in Ljung (1999). For more references, see "Reading More About System Identification" on page 1-22.) The process operates much like a common hand-held hair dryer. Air is blown through a tube after being heated at the inlet to the tube. The input to the process is the power applied to a mesh of resistor wires that constitutes the heating device. The output of the process is the air temperature at the outlet, measured in volts by a thermocouple sensor.

One thousand input-output data points were collected from the process as the input was changed in a random fashion between two levels. The sampling interval is 80 ms. The data was loaded into MATLAB in ASCII form and is now stored as the vectors `y2` (output) and `u2` (input) in the file `dryer2.mat`.

1. Load the data.

   ```
   load dryer2
   ```

2. It contains the input vector `u2`, the output vector `y2`. Now form the data object.

   ```
   dry = iddata(y2,u2,0.08);
   ```

3. To get information about the data, just type the name.

   ```
   dry
   ```

4. To get an overview of all the information contained in the `iddata` object `dry`, type

   ```
   get(dry)
   ```

5 For better bookkeeping, give names to input and outputs.

```
dry.InputName = 'Power';

dry.OutputName = 'Temperature';
```

6 Select the 300 first values for building a model.

```
ze = dry(1:300);
```

7 Plot the interval from sample 200 to 300.

```
plot(ze(200:300)),
```

8 Remove the constant levels and make the data zero-mean.

```
ze = detrend(ze);
```

9 First estimate the impulse response of the system by correlation analysis to get some idea of time constants and the like.

```
impulse(ze,'sd',3)
```

This gives a plot with dash-dotted lines marking a confidence region corresponding to three standard deviations (ca 99.9%). From this it is easy to see if there is a time delay in the system.

Getting Started

The simplest way to get started is to build a state-space model where the order is automatically determined, using a prediction error method.

```
m1 = pem(ze)
```

When the calculations are finished, a display of the basic information about `m1` is shown. Any time you type `m1`, this display is shown. Typing `present(m1)` gives some more information about the model, including uncertainties.

To retrieve the properties of this model you could, for example, find the `A` matrix of the state space representation by

```
A = m1.a
```

`m1` is a model object, and

```
get(m1)
```

gives a list of all information stored in the model.

`m1.EstimationInfo` or `m1.es` for short gives information about the estimation process, loss functions, etc.

How Good Is the Model?

How good is this model? One way to find out is to simulate it and compare the model output with measured output. Select a portion of the original data that was not used to build the model, for example, from sample 800 to 900.

```
zv = dry(800:900);
zv = detrend(zv);
compare(zv,m1);
```

The Bode plot of the model is obtained by

```
bode(m1)
```

An alternative is to consider the Nyquist plot and mark uncertainty regions at certain frequencies with ellipses, corresponding to three standard deviations.

```
nyquist(m1,'sd',3)
```

You can also compare the step response of the model with one that is directly computed from data (`ze`) in a nonparametric way.

```
step(m1,ze)
```

To study a model with prescribed structure, compute a difference equation model with two poles, one zero, and three delays.

```
m2 = arx(ze,[2 2 3])
```

This produces a model of the form

$$y(t) + a_1 y(t-T) + a_2 y(t-2T) = b_1 u(t-3T) + b_2 u(t-4T)$$

where T is the sampling interval (here 0.08 second). This model, known as an ARX model, tries to explain or compute the value of the output at time t, given previous values of y and u. To compare its performance on validation data with `m1`, type

```
compare(zv,m1,m2);
```

Compare and Plot

Compute and plot the poles and zeros of the models.

```
pzmap(m1,m2)
```

The uncertainties of the poles and zeros can also be plotted.

```
pzmap(m1,m2,'sd',3), % '3' denotes the number of standard
deviations
```

Estimate the frequency response by a nonparametric spectral analysis method.

```
gs = spa(ze);
```

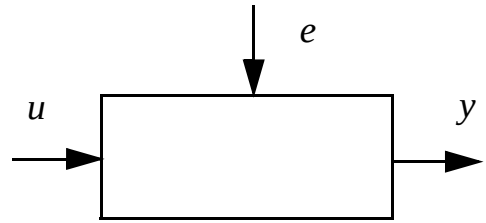
Compare with the frequency functions from the parametric models.

```
bode(m1,m2,gs)
```

The System Identification Problem

This section discusses basic ways to describe linear dynamic systems and the most important methods for estimating such models.

Impulse Responses, Frequency Functions, and Spectra



The basic input-output configuration is depicted in the figure above. Assuming unit sampling interval, there is an input signal

$$u(t); \qquad t = 1, 2, \ldots, N$$

and an output signal

$$y(t); \qquad t = 1, 2, \ldots, N$$

Assuming the signals are related by a linear system, the relationship can be written

$$y(t) = G(q)u(t) + v(t) \tag{3-1}$$

where q is the shift operator and $G(q)u(t)$ is short for

$$G(q)u(t) = \sum_{k=1}^{\infty} g(k)u(t-k) \tag{3-2}$$

and

$$G(q) = \sum_{k=1}^{\infty} g(k)q^{-k}; \qquad q^{-1}u(t) = u(t-1) \tag{3-3}$$

The numbers $\{g(k)\}$ are called the *impulse response* of the system. Clearly, $g(k)$ is the output of the system at time k if the input is a single (im)pulse at time zero. The function $G(q)$ is called the *transfer function* of the system. This function evaluated on the unit circle ($q = e^{i\omega}$) gives the *frequency function* (or *frequency-response function*).

$$G(e^{i\omega}) \tag{3-4}$$

In (Equation 3-1) $v(t)$ is an additional, unmeasurable disturbance (noise). Its properties can be expressed in terms of its (power) spectrum

$$\Phi_v(\omega) \tag{3-5}$$

which is defined by

$$\Phi_v(\omega) = \sum_{\tau=-\infty}^{\infty} R_v(\tau)e^{-i\omega\tau} \tag{3-6}$$

where $R_v(\tau)$ is the covariance function of $v(t)$

$$R_v(\tau) = Ev(t)v(t-\tau) \tag{3-7}$$

and E denotes mathematical expectation. Alternatively, the disturbance $v(t)$ can be described as filtered white noise

$$v(t) = H(q)e(t) \tag{3-8}$$

where $e(t)$ is white noise with variance λ and

$$\Phi_v(\omega) = \lambda\left|H(e^{i\omega})\right|^2 \tag{3-9}$$

(Equation 3-1) and (Equation 3-8) together give a *time-domain description* of the system

$$y(t) = G(q)u(t) + H(q)e(t) \tag{3-10}$$

where G is the *transfer function* of the system. (Equation 3-4) and (Equation 3-5) constitute a *frequency-domain description*.

$$G(e^{i\omega}); \qquad \Phi_v(\omega) \tag{3-11}$$

The impulse response (Equation 3-3) and the frequency-domain description (Equation 3-11) are called *nonparametric model descriptions* because they are not defined in terms of a finite number of parameters. The basic description (Equation 3-10) also applies to the multivariable case, that is, to systems with several (say *nu*) input signals and several (say *ny*) output signals. In that case $G(q)$ is an *ny*-by-*nu* matrix while $H(q)$ and $\Phi_v(\omega)$ are *ny*-by-*ny* matrices.

Polynomial Representation of Transfer Functions

Rather than specifying the functions *G* and *H* in (Equation 3-10) in terms of functions of the frequency variable ω, you can describe them as rational functions of q^{-1} and specify the numerator and denominator coefficients in some way.

A commonly used parametric model is the ARX model that corresponds to

$$G(q) = q^{-nk} \cdot \frac{B(q)}{A(q)}; \qquad H(q) = \frac{1}{A(q)} \tag{3-12}$$

where *B* and *A* are polynomials in the delay operator q^{-1}.

$$\begin{aligned} A(q) &= 1 + a_1 q^{-1} + \ldots\ldots + a_{na} q^{-na} \\ B(q) &= b_1 + b_2 q^{-1} + \ldots\ldots + b_{nb} q^{-nb+1} \end{aligned} \tag{3-13}$$

Here, the numbers *na* and *nb* are the orders of the respective polynomials. The number *nk* is the number of delays from input to output. The model is usually written

$$A(q)y(t) = B(q)u(t-nk) + e(t) \tag{3-14}$$

or explicitly

$$\begin{aligned} &y(t) + a_1 y(t-1) + \ldots\ldots + a_{na} y(t-na) = \\ &b_1 u(t-nk) + b_2 u(t-nk-1) + \ldots\ldots + b_{nb} u(t-nk-nb+1) + e(t) \end{aligned} \tag{3-15}$$

Note that (Equation 3-14) and (Equation 3-15) apply also to the multivariable case, with *ny* output channels and *nu* input channels. Then $A(q)$ and the coefficients a_i become *ny*-by-*ny* matrices, and $B(q)$ and the coefficients b_i become *ny*-by-*nu* matrices.

Another very common, and more general, model structure is the ARMAX structure

$$A(q)y(t) = B(q)u(t-nk) + C(q)e(t) \quad \textbf{(3-16)}$$

Here, $A(q)$ and $B(q)$ are as in (Equation 3-13), while

$$C(q) = 1 + c_1 q^{-1} + \ldots + c_{nc} q^{-nc}$$

An *output-error* (OE) structure is obtained as

$$y(t) = \frac{B(q)}{F(q)} u(t-nk) + e(t) \quad \textbf{(3-17)}$$

with

$$F(q) = 1 + f_1 q^{-1} + \ldots + f_{nf} q^{-nf}$$

The so-called *Box-Jenkins* (BJ) model structure is given by

$$y(t) = \frac{B(q)}{F(q)} u(t-nk) + \frac{C(q)}{D(q)} e(t) \quad \textbf{(3-18)}$$

with

$$D(q) = 1 + d_1 q^{-1} + \ldots + d_{nd} q^{-nd}$$

All these models are special cases of the general parametric model structure.

$$A(q)y(t) = \frac{B(q)}{F(q)} u(t-nk) + \frac{C(q)}{D(q)} e(t) \quad \textbf{(3-19)}$$

The variance of the white noise $\{e(t)\}$ is assumed to be λ.

Within the structure of (Equation 3-19), virtually all the usual linear black-box model structures are obtained as special cases. The ARX structure is obviously obtained for $nc = nd = nf = 0$. The ARMAX structure corresponds to $nf = nd = 0$. The ARARX structure (or the *generalized least squares model*) is obtained for $nc = nf = 0$, while the ARARMAX structure (or *extended matrix model*) corresponds to $nf = 0$. The output-error model is obtained with $na = nc = nd = 0$, while the Box-Jenkins model corresponds to $na = 0$. (See Section 4.2 in Ljung (1999) for a detailed discussion.)

The same type of models can be defined for systems with an arbitrary number of inputs. They have the form

$$A(q)y(t) = \frac{B_1(q)}{F_1(q)}u_1(t-nk_1) + \ldots + \frac{B_{nu}(q)}{F_{nu}(q)}u_{nu}(t-nk_{nu}) + \frac{C(q)}{D(q)}e(t) \quad \textbf{(3-20)}$$

State-Space Representation of Transfer Functions

A common way of describing linear systems is to use the *state-space form*.

$$\begin{aligned} x(t+1) &= Ax(t) + Bu(t) \\ y(t) &= Cx(t) + Du(t) + v(t) \end{aligned} \quad \textbf{(3-21)}$$

Here the relationship between the input $u(t)$ and the output $y(t)$ is defined via the *nx*-dimensional *state vector* $x(t)$. In transfer function form (Equation 3-21) corresponds to (Equation 3-1) with

$$G(q) = C(qI_{nx} - A)^{-1}B + D \quad \textbf{(3-22)}$$

Here I_{nx} is the *nx*-by-*nx* identity matrix. Clearly (Equation 3-21) can be viewed as one way of parameterizing the transfer function: With (Equation 3-22), $G(q)$ becomes a function of the elements of the matrices A, B, C, and D.

To further describe the character of the noise term $v(t)$ in (Equation 3-21), a more flexible *innovations* form of the state-space model can be used.

$$\begin{aligned} x(t+1) &= Ax(t) + Bu(t) + Ke(t) \\ y(t) &= Cx(t) + Du(t) + e(t) \end{aligned} \quad \textbf{(3-23)}$$

This is equivalent to (Equation 3-10) with $G(q)$ given by (Equation 3-22) and $H(q)$ by

$$H(q) = C(qI_{nx} - A)^{-1}K + I_{ny} \quad \textbf{(3-24)}$$

Here *ny* is the dimension of $y(t)$ and $e(t)$.

It is often possible to set up a system description directly in the innovations form (Equation 3-23). In other cases, it might be preferable to describe first the nature of disturbances that act on the system. That leads to a stochastic state-space model

$$\begin{aligned} x(t+1) &= Ax(t) + Bu(t) + w(t) \\ y(t) &= Cx(t) + Du(t) + e(t) \end{aligned} \quad \textbf{(3-25)}$$

where $w(t)$ and $e(t)$ are stochastic processes with certain covariance properties. If you neglect transients and consider only the input-output properties, (Equation 3-25) is equivalent to (Equation 3-23) if the matrix K is chosen as the steady-state *Kalman gain*. How to compute K from (Equation 3-25) is described in the Control System Toolbox documentation.

Continuous-Time State-Space Models

It is often easier to describe a system from physical modeling in terms of a continuous-time model. The reason is that most physical laws are expressed in continuous time as differential equations. Therefore, physical modeling typically leads to state-space descriptions like

$$\begin{aligned} \dot{x}(t) &= Fx(t) + Gu(t) \\ y(t) &= Hx(t) + Du(t) + v(t) \end{aligned} \tag{3-26}$$

Here, $\dot{x}$ means the time derivative of x. If the input is piecewise constant over time intervals $kT \le t < (k+1)T$, then the relationship between $u[k] = u(kT)$ and $y[k] = y(kT)$ can be exactly expressed by (Equation 3-21) by taking

$$A = e^{FT}; \qquad B = \int_0^T e^{F\tau} G d\tau; \qquad C = H \tag{3-27}$$

and associating $v(tT)$ with $y[t]$, etc. If you start with a continuous-time innovations form

$$\begin{aligned} \dot{x}(t) &= Fx(t) + Gu(t) + \tilde{K}e(t) \\ y(t) &= Hx(t) + Du(t) + e(t) \end{aligned} \tag{3-28}$$

the discrete-time counterpart is given by (Equation 3-23) where the relationships (Equation 3-27) still hold. The exact connection between $\tilde{K}$ and K is somewhat more complicated, though. An ad hoc solution is to use

$$K = \int_0^T e^{F\tau} \tilde{K} d\tau; \tag{3-29}$$

in analogy with G and B. This is a good approximation for short sampling intervals T.

Estimating Impulse Responses

Consider the descriptions (Equation 3-1) and (Equation 3-2). To directly estimate the impulse response coefficients, also in the multivariable case, it is suitable to define a high-order Finite Impulse Response (FIR) model

$$y(t) = g(0)u(t) + g(1)u(t-1) + \ldots + g(n)u(t-n) \qquad \textbf{(3-30)}$$

and estimate the g-coefficients by the linear least squares method. In fact, to check whether there are noncausal effects from input to output, for example, due to feedback from y in the generation of u (closed loop data), g for negative lags can also be estimated.

$$\begin{aligned} y(t) = {} & g(-m)u(t+m) + \ldots + g(-1)u(t+1) + g(0)u(t) + \\ & g(1)u(t-1) + \ldots + g(n)u(t-n) \end{aligned} \qquad \textbf{(3-31)}$$

If u is white noise, the impulse response coefficients will be correctly estimated, even if the true dynamics from u to y are more complicated than these models. Therefore it is natural to filter both the output and the input through a filter that makes the input sequence as white as possible before estimating the g. This is the essence of *correlation analysis* for estimating impulse responses.

Estimating Spectra and Frequency Functions

This section describes methods that estimate the frequency functions and spectra (Equation 3-11) directly. The cross-covariance function $R_{yu}(\tau)$ between $y(t)$ and $u(t)$ is defined as $Ey(t+\tau)u(t)$ analogously to (Equation 3-7). Its Fourier transform, the cross spectrum $\Phi_{yu}(\omega)$, is defined analogously to (Equation 3-6). Provided that the input $u(t)$ is independent of $v(t)$, the relationship (Equation 3-1) implies the following relationships between the spectra.

$$\begin{aligned} \Phi_y(\omega) &= \left|G(e^{i\omega})\right|^2 \Phi_u(\omega) + \Phi_v(\omega) \\ \Phi_{yu}(\omega) &= G(e^{i\omega})\Phi_u(\omega) \end{aligned} \qquad \textbf{(3-32)}$$

By estimating the various spectra involved, you can estimate the frequency function and the disturbance spectrum as follows:

Form estimates of the covariance functions (as defined in (Equation 3-7)) $\hat{R}_y(\tau)$, $R_{yu}(\tau)$, and $\hat{R}_u(\tau)$, using

$$\hat{R}_{yu}(\tau) = \frac{1}{N}\sum_{t=1}^{N} y(t+\tau)u(t) \tag{3-33}$$

and analog expressions for the others. Then, form estimates of the corresponding spectra

$$\hat{\Phi}_y(\omega) = \sum_{\tau=-M}^{M} \hat{R}_y(\tau)W_M(\tau)e^{-i\omega\tau} \tag{3-34}$$

and analogously for Φ_u and Φ_{yu}. Here $W_M(\tau)$ is the so-called *lag window* and M is the width of the lag window. The estimates are then formed as

$$\hat{G}_N(e^{i\omega}) = \frac{\hat{\Phi}_{yu}(\omega)}{\hat{\Phi}_u(\omega)}; \qquad \hat{\Phi}_v(\omega) = \hat{\Phi}_y(\omega) - \frac{\left|\hat{\Phi}_{yu}(\omega)\right|^2}{\hat{\Phi}_u(\omega)} \tag{3-35}$$

This procedure is known as *spectral analysis*. (See Chapter 6 in Ljung (1999).)

Estimating Parametric Models

Given a description (Equation 3-10) and having observed the input-output data u, y, the (prediction) errors $e(t)$ in (Equation 3-10) can be computed as

$$e(t) = H^{-1}(q)[y(t) - G(q)u(t)] \tag{3-36}$$

These errors are, for given data y and u, functions of G and H. These in turn are parameterized by the polynomials in (Equation 3-14) through (Equation 3-19) or by entries in the state-space matrices defined in (Equation 3-26) through (Equation 3-29). The most common parametric identification method is to determine estimates of G and H by minimizing

$$V_N(G,H) = \sum_{t=1}^{N} e^2(t) \tag{3-37}$$

that is

$$[\hat{G}_N, \hat{H}_N] = argmin \sum_{t=1}^{N} e^2(t) \quad \textbf{(3-38)}$$

This is called a *prediction error method*. For Gaussian disturbances it coincides with the maximum likelihood method. (See Chapter 7 in Ljung (1999).)

A somewhat different philosophy can be applied to the ARX model (Equation 3-14). By forming filtered versions of the input

$$N(q)s(t) = M(q)u(t) \quad \textbf{(3-39)}$$

and by multiplying (Equation 3-14) with $s(t-k)$, $k = 1, 2, \ldots, na$ and $u(t-nk+1-k)$, $k = 1, 2, \ldots, nb$ and summing over t, the noise in (Equation 3-14) can be correlated out and solved for the dynamics. This gives the *instrumental variable* method, and $s(t)$ are called the instruments. (See Section 7.6 in Ljung (1999).)

Subspace Methods for Estimating State-Space Models

The state-space matrices A, B, C, D, and K in (Equation 3-23) can be estimated directly, without first specifying any particular parameterization by efficient *subspace methods*. The idea behind this can be explained as follows: If the sequence of state vectors $x(t)$ were known, together with $y(t)$ and $u(t)$, (Equation 3-23) would be a linear regression, and C and D could be estimated by the least squares method. Then $e(t)$ could be determined, and treated as a known signal in (Equation 3-23), which then would be another linear regression model for A, B, and K. (One could also treat (Equation 3-21) as a linear regression for A, B, C, and D with $y(t)$ and $x(t+1)$ as simultaneous outputs, and find the joint process and measurement noises as the residuals from this regression. The Kalman gain K could then be computed from the Riccati equation.) Thus, once the states are known, the estimation of the state-space matrices is easy.

How to find the states $x(t)$? All states in representations like (Equation 3-23) can be formed as linear combinations of the k-step-ahead predicted outputs ($k = 1, 2, \ldots, n$). It is thus a matter of finding these predictors, and then selecting a basis among them. The subspace methods form an efficient and numerically reliable way of determining the predictors by projections directly

on the observed data sequences. See Sections 7.3 and 10.6 in Ljung (1999). For more details, see the references under `n4sid` in the reference pages.

The advice Command

A general command, `advice`, can be applied to any estimated model and to any data set,

```
advice(model)
advice(data)
```

to provide the user with information about the quality of the model and characteristics, possibilities, and fallacies for the data set.

Data Representation and Nonparametric Model Estimation

This and the following sections introduce the basic functions in the System Identification Toolbox. Not all of the options available when using the functions are described here; see Chapter 4, "Functions — By Category" and the online Help facility.

Data Representation

The observed output and input signals, $y(t)$ and $u(t)$, are represented as *column vectors* `y` and `u`. Row `k` corresponds to sample number k. For multivariable systems, each input (output) component is represented as a column vector, so that `u` becomes an N-by-nu matrix (N = number of sampled observations, nu = number of input channels). The output-input data is collectively represented in the `iddata` format. This is the basic object for dealing with signals in the toolbox. It is used by most of the commands. Create it using

```
Data = iddata(y,u,Ts)
```

where `y` is a column vector or an N-by-ny matrix. The columns of `y` correspond to the output channels. Similarly `u` is a column vector or an N-by-nu matrix containing the signals of the input channels. `Ts` is the sampling interval. This construction is sufficient for almost all purposes.

The data is then plotted by `plot(Data)` and portions of the data record are selected, as in

```
ze = Data(1:300)
```

You can retrieve the signals in the output channels using `Data.OutputData` or, for short, `Data.y`. Similarly you can obtain the input signals using `Data.InputData` or `Data.u`.

For a time series (no input channels) use `Data = iddata(y)`, or let `u = [ ]`. An `iddata` object can also contain just an input if you let `y = [ ]`.

You can change the sampling interval by using `set(Data,'Ts',0.3)` or, more simply, by

```
Data.Ts = 0.3
```

More details about the `iddata` object are given at the end of this section.

Correlation Analysis

The correlation analysis procedure described in "Estimating Impulse Responses" on page 3-15 is implemented in the function `impulse`.

```
impulse(Data)
```

This function plots the estimated impulse response. Adding an argument `'sd'` as in

```
impulse(Data,'sd',3)
```

also marks a confidence region corresponding to (in this case) three standard deviations. The result can be stored and replotted.

```
ir = impulse(Data)
impulse(ir,'sd',3)
```

An alternative is the command `step` that plots the step response, calculated from the impulse estimate.

```
step(Data)
```

Spectral Analysis

The function `spa` performs spectral analysis according to the procedure in (Equation 3-35) through (Equation 3-37).

```
g = spa(Data)
```

Here `Data` contains the output-input data in the `iddata` object as above. `g` is returned as an `idfrd` (identified frequency response data) model object that contains the estimated frequency function G_N and the estimated disturbance spectrum Φ_v in (Equation 3-37), as well as estimated uncertainty covariances. The `idfrd` object is described in the `idfrd` reference page, but for normal use you do not have to bother about these details. The frequency function, or *frequency response*, G in `g` can be graphed by the function `bode`, `ffplot`, or `nyquist`. The noise spectrum is retrieved by `g('n')` (`'n'` for noise) so

```
g = spa(Data)
bode(g)
bode(g('n'))
```

performs the spectral analysis, and plots first G and then Φ_v. The `bode` function gives logarithmic amplitude and frequency scales (in rad/s) and linear

phase scale, while `ffplot` gives linear frequency scales (in Hz). You can display the uncertainty of the estimates by adding the argument `'sd'`, as in

```
bode(g,'sd',3)
```

which displays, by dash-dotted lines, a confidence region around the estimate that corresponds (in this case) to three standard deviations. Adding an argument `'fill'` shows the uncertainty region as a filled region instead.

```
bode(g,'sd',3,'fill')
```

Similarly,

```
nyquist(g)
```

gives a Nyquist plot of the frequency function, that is, a plot of the real part versus the imaginary part of G.

If `Data = y` is a time series, that is, `Data` has no input channel, `spa` returns an estimate of the spectrum of that signal.

```
g= spa(y)
ffplot(g)
```

In the computations (Equation 3-35) through (Equation 3-37), `spa` uses as a lag window the Hamming window for $W(\tau)$ with a default length M equal to the minimum of 30 and a tenth of the number of data points. You can change this window size M to an arbitrary number using

```
g = spa(Data,M)
```

The rule is that as `M` increases, the estimated frequency functions show sharper details, but are also more affected by random disturbances. A typical sequence of commands that test different window sizes is

```
g10 = spa(Data,10)
g25 = spa(Data,25)
g50 = spa(Data,50)
bode(g10, g25, g50)
```

An empirical transfer function estimate is obtained as the ratio of the output and input Fourier transforms with

```
g = etfe(Data)
```

This can also be interpreted as the spectral analysis estimate for a window size that is equal to the data length. For time series, `etfe` gives the *periodogram* as a spectral estimate. The function also allows some smoothing of the crude estimate; it can be a good alternative for signals and systems with sharp resonances. See Chapter 4, "Functions — By Category" for more information.

Estimation of spectra and frequency functions involves a tradeoff between *resolution* and *noise sensitivity*. By resolution is meant the finest details (in rad/s) that can be distinguished in the estimate, while noise sensitivity describes how disturbances of different kinds give high variability in the estimates. The number M mentioned above is a way to control this tradeoff globally over the frequency range.

A useful complement to `etfe` and `spa` is the possibility of having frequency-dependent resolution, using the command `spafdr`,

```
g = spafdr(Data)
g = spafrd(Data,Res,Freqs)
```

with the possibility of defining both the frequencies `Freqs` for which the estimate should be formed and the resolution `Res` for the different frequencies. See the `spafdr` reference page for more details.

Frequency Domain Data

The `iddata` object can also represent frequency-domain data, that is, input and output signals that are Fourier transforms of time-domain signals. Such data sets are useful in many contexts. You create a frequency-domain data set by

```
Data = iddata(Y,U,Ts,'Domain,'Frequency','freq',W)
```

where `Y` and `U` are the output and input Fourier transforms (`N`-by-`ny` and `N`-by-`nu` complex-valued matrices) and `W` is the vector of associated frequencies. That means that `Y(kf,ky)` is the frequency component of output number `ky` at frequency `W(kf)`. Frequency-domain data can also easily be constructed from time-domain data, as in

```
dataf = fft(data)
```

A further way to handle frequency-domain information for model estimation is to define a frequency response data object (IDFRD) that contains the frequency-response data of a system, as in Equation 3-11:

```
datfr = idfrd(G,W,Ts)
```

```
datfr = idfrd(G,W,Ts,'SpectrumData',Phiv)
```

Here `G` is the frequency response function, `W` is the vector of frequencies, and `Ts` is the sampling interval. Optionally, you can also include the additive output spectrum `Phiv` = $\Phi_v(\omega)$.

You can also create a frequency-response data object from a model or from data by

```
datafr = idfrd(model)
datafr = spafdr(Data)
```

(Compare the techniques on page 3-20.) While `datafr` can be seen as a nonparametric model of the system, it can also be seen as a more compact way of representing the data `Data`. This representation can be used to further estimate parametric models. Also, in many applications it is common to use frequency analyzers for data acquisition. They deliver data in the frequency-function form rather than as separate input and output signals, in a much more compact form.

More details of this are given on the reference pages for `iddata` and `idfrd`. The main message here is that the handling of data in time and frequency domain is essentially transparent. All estimation and representation commands that apply to time-domain data can also be used with the same syntax for frequency-domain data.

Two differences can be noted:

- Noise models cannot be estimated from frequency-domain data.
- Frequency-domain data can handle representation of time-continuous signals (`Ts = 0`). This means that `Y` and `U` are the continuous-time Fourier transforms given at a finite number of frequencies.

More on the Data Representation in iddata

Some Bookkeeping Facilities

The input and output channels are given default names like `y1`, `y2`, `u1`, `u2`, etc. You can set the channel names using

```
set(Data,'InputName',{'Voltage','Current'},'OutputName','Tempera
ture')
```

(two inputs and one output in this example) and these names will then follow the object and appear in all plots. The names are also inherited by models that are estimated from the data.

Similarly, you can specify channel units using the properties `OutputUnit` and `InputUnit`. These units, when specified, are used in plots.

The time points associated with the data samples are determined by the sampling interval `Ts` and the time of the first sample, `Tstart`.

```
Data.Tstart = 24
```

The actual time-point values are given by the property `SamplingInstants`, as in

```
plot(Data.sa,Data.u)
```

for a plot of the input with correct time points. Autofill is used for all properties, and they are case insensitive. For easy writing, `'u'` is synonymous with `'Input'` and `'y'` with `'Output'` when you are referring to the properties.

Manipulating Channels

An easy way to set and retrieve channel properties is to use subscripting. The subscripts are defined as

```
Data(samples,outputs,inputs)
```

so `Dat(:,3,:)` is the data object obtained from `Dat` by keeping all input channels, but only output channel 3. (Trailing colons can be omitted, so `Dat(:,3,:) = Dat(:,3)`.)

You can also retrieve the channels by their names, so that

```
Dat(:,{'speed','flow'},[ ])
```

is the data object where the indicated output channels have been selected and no input channels are selected.

Moreover,

```
Dat1(101:200,[3 4],[1 3]) = Dat2(1001:1100,[1 2],[6 7])
```

will change samples 101 to 200 of output channels 3 and 4 and input channels 1 and 3 in the `iddata` object `Dat1` to the indicated values from `iddata` object `Dat2`. The names and units of these channels are then also changed accordingly.

To add new channels, use horizontal concatenation of `iddata` objects.

```
Dat =[Dat1, Dat2];
```

See "Adding Channels" on page 3-27 or add the data record directly, so that

```
Dat.u(:,5) = u
```

adds a fifth input to `Dat`.

Nonequal Sampling

The property `SamplingInstants` gives the sampling instants of the data points. It can always be retrieved by `get(Dat,'SamplingInstants')` (or `Dat.s`) and is then computed from `Dat.Ts` and `Dat.Tstart`. `SamplingInstants` can also be set to an arbitrary vector of the same length as the data, so that nonequal sampling can be handled. `Ts` is then automatically set to [ ]. Most of the estimation routines, however, do not handle unequally sampled data.

Multiple Experiments

The `iddata` object can also store data from separate experiments. The property `ExperimentName` is used to separate the experiments. The number of data as well as the sampling properties can vary from experiment to experiment, but the input and output channels must be the same. (Use `NaN`s to fill unmeasured channels in certain experiments.) The data records will be cell arrays where the cells contain data from each experiment.

You can define multiple experiments directly by letting the `'y'` and `'u'` properties as well as `'Ts'` and `'Tstart'` be cell arrays.

It is normally easier to create multiple-experiment data by merging experiments, as in

```
Dat = merge(Dat1,Dat2)
```

See the `merge (iddata)` reference page. Storing multiple experiments as one `iddata` object can be very useful to handle experimental data that has been collected on different occasions, or when a data set has been split up to remove bad portions of the data. All the toolbox's routines accept multiple-experiment data.

You can retrieve experiments using the command `getexp`, as in `getexp(Dat,3)` or `getexp(Dat,'Period1')`. You can also set and retrieve them by subscripting with a fourth index: `Dat(:,:,:,3)}` is experiment number 3 and

`Dat(:,:,:,{'Day1','Day4'})` retrieves the two experiments with the indicated names.

The subscripting can be combined: `Dat(1:100,[2,3],[4:8],3)` gives the 100 first samples of output channels 2 and 3 and input channels 4 to 8 of experiment number 3. You can also use subscripting for subassignment:

```
Dat(:,:,:,''Run4') = Dat2
```

adds the data in `Dat2` as a new experiment with name `'Run4'`. See `iddemo #9` for an illustration of how multiple experiments can be used.

iddata Properties

Type `get(Dat)` or see the `iddata` reference page for a complete list of `iddata` properties.

Subreferencing

The samples, outputs, and input channels can be referenced according to

```
Data(samples,outputs,inputs)
```

Use a colon (:) to denote all samples/channels and the empty matrix ([ ]) to denote no samples/channels. The channels can be referenced by number or by name. For several names you must use a cell array.

```
Dat2 = Dat(:,'y3',{'u1','u4'})
Dat2 = Dat(:,3,[1 4])
```

Logical expressions also work.

```
Dat3 = Dat2(Dat2.sa>1.27&Dat2.sa<9.3)
```

selects the samples with time marks between 1.27 and 9.3.

Any subreferenced variable can also be assigned.

```
Data(1:10,1,1) = Dat1(101:110,2,3)
```

Adding Channels

```
Dat = [Dat1,Dat2,...,DatN]
```

creates an `iddata` object `Dat`, consisting of the input and output channels in `Dat1,... DatN`. Default channel names (`'u1'`, `'u2'`, `'y1'`, `'y2'`, etc.) are changed so that overlaps in names are avoided, and the new channels are added.

If `Datk` contains channels with user-specified names that are already present in the channels of `Datj, j<k`, these new channels are ignored.

Adding Samples

```
Dat = [Dat1;Dat2;... ;DatN]
```

creates an `iddata` object `Dat` whose signals are obtained by stacking those of `Datk` on top of each other, that is,

```
Dat.y = [Dat1.y;Dat2.y; ... DatN.y]
```

and similarly for the inputs. The `Datk` objects must all have the same number of channels and experiments.

Parametric Model Estimation

The System Identification Toolbox contains several functions for parametric model estimation. They all share the same command structure.

```
m = function(Data,modstruc)
m =   ...
function(Data,modstruc,'Property1',Value1,...'PropertyN',ValueN)
```

The argument `Data` is an `iddata` object that contains the output and input data sequences, while `modstruc` specifies the particular structure of the model to be estimated. The resulting estimated model is contained in `m`. It is a model object that stores various information. The model objects will be described in "Defining Model Structures" on page 3-39, but for most use of the toolbox, you do not have to consider the details of these objects. Just typing the model name

```
m
```

will give a concise display of the model. The command

```
present(m)
```

gives some more details, while

```
get(m)
```

gives a complete list of the model's properties. The property values can be easily retrieved just by dot-referencing. For example,

```
m.par
```

retrieves the estimated parameters.

In the function call `(...,'Property1', Value1,...,'PropertyN',ValueN)` is a list of properties that can be assigned to affect the model structure as well as the estimation algorithm. A list of typical properties is given at the end of this section. The model `m` is also immediately prepared for displaying and analyzing its characteristics as well as for transforming it to other representations, as in

```
bode(m)
compare(Data,m)
[A,B,C,D, K] = ssdata(m)
```

See "Examining Models" on page 3-57 for a detailed discussion of these possibilities.

In the following, `Data` denotes an `iddata` object that contains the input output data as described in the previous section. It can also just contain an output signal, that is, a time series.

ARX Models

To estimate the parameters a_i and b_i of the ARX model (Equation 3-14), use the function `arx`.

```
m  = arx(Data,[na nb nk])
```

Here `na`, `nb`, and `nk` are the corresponding orders and delays in (Equation 3-15) that define the exact model structure. The function `arx` implements the least squares estimation method, using QR-factorization for overdetermined linear equations.

An alternative is to use the instrumental variable (IV) method described in connection with (Equation 3-39). This is obtained with

```
m = iv4(Data,[na nb nk])
```

which gives an automatic (and approximately optimal) choice of the filters N and M in (Equation 3-39). (See the procedure (15.21)-(15.26) in Ljung (1999).)

Both `arx` and `iv4` are applicable to arbitrary multivariable systems. If you have *ny* outputs and *nu* inputs, the orders are defined accordingly: `na` is an *ny*-by-*ny* matrix whose *i-j*th entry gives the order of the polynomial that relates past values of y_j to the current value of y_i. In other words, the past values of y_j up to $y_j(t - na(i,j))$ are used when predicting $y_i(t)$. Similarly, the *i-j* entries of the *ny*-by-*nu* matrices `nu` and `nk`, respectively, give the order and delay from input number *j* when predicting output number *i*. (See "Multivariable ARX Models: the idarx Model" on page 3-43 and Chapter 4, "Functions — By Category" for exact details.)

AR Models

For a single output signal $y(t)$, the counterpart of the ARX model is the AR model.

$$A(q)y(t) = e(t) \qquad \textbf{(3-40)}$$

The `arx` command also covers this special case:

```
m = arx(y,na)
```

but for scalar signals more options are offered by the command

```
m = ar(y,na)
```

which has an option that allows you to choose the algorithm from a group of several popular techniques for computing the least squares AR model. Among these are Burg's method, a geometric lattice method, the Yule-Walker approach, and a modified covariance method. (See Chapter 4, "Functions — By Category" for details.) The counterpart of the `iv4` command is

```
m = ivar(y,na)
```

which uses an instrumental variable technique to compute the AR part of a time series.

General Polynomial Black-Box Models

Based on the prediction error method (Equation 3-38), you can construct models of basically any structure. For the general model (Equation 3-19), there is the function

```
m = pem(Data,nn)
```

where `nn` gives all the orders and delays.

```
nn = [na nb nc nd nf nk]
```

The nonzero orders of the model can also be defined as property name/property value pairs, as in

```
m = pem(Data,'na',na,'nb',nb,'nc',nc,'nk',nk)
```

The input parameters are defined in "Polynomial Representation of Transfer Functions" on page 3-11. The `pem` command covers all cases of black-box linear system models. For the common special cases,

```
m = armax(Data,[na nb nc nk])
m = oe(Data,[nb nf nk])
m = bj(Data,[nb nc nd nf nk])
```

can be used. These handle the model structures (Equation 3-16), (Equation 3-17), and (Equation 3-18), respectively.

All the routines also cover single-output, multiinput systems of the type

$$A(q)y(t) = \frac{B_1(q)}{F_1(q)}u_1(t-nk_1) + \ldots + \frac{B_{nu}(q)}{F_{nu}(q)}u_{nu}(t-nk_{nu}) + \frac{C(q)}{D(q)}e(t) \quad \textbf{(3-41)}$$

where `nb`, `nf`, and `nk` are row vectors of the same lengths as the number of input channels containing each of the orders and delays:

```
nb = [nb1 ...  nbnu];
nf = [nf1 ...  nfnu];
nk = [nk1 ...  nknu];
```

These parameter estimation routines require an iterative search for the minimum of the function (Equation 3-39). This search uses a special startup procedure based on least squares and instrumental variables (the details are given as Equation (10.79) in Ljung (1999)). From the initial estimate, a Gauss-Newton minimization procedure is carried out until the norm of the Gauss-Newton direction is less than a certain tolerance. See Ljung (1999), Section 11.2, or Dennis and Schnabel (1983) for details. See also "Optional Variables" on page 3-33 on optional variables associated with the search.

The estimation routines also return the estimated covariance matrix of the estimated parameter vector as part of `m`. This reflects the reliability of the estimates. The covariance matrix estimate is computed under the assumption that it is possible to obtain a true description in the given structure.

You can also start the routines `pem`, `armax`, `oe`, and `bj` at any initial value `mi` that is a model object, by replacing `nn` by `mi`. For example,

```
m = pem(Data,mi)
```

While the search is typically initialized using the built-in startup procedure giving just orders and delays (as described above), the ability to force a specific initial condition is useful in several contexts. Some examples are mentioned in "Initial Parameter Values" on page 3-99.

Information about how the minimization progresses can be supplied to the MATLAB Command Window by the property `trace`. See the list in "Properties That Apply to Estimation Methods Using Iterative Search for Minimizing a Criterion" on page 3-36.

Process Models

For process control applications, often simple continuous-time models are used, consisting of static gain, time constants, and a possible dead time (time delay). Such models are estimated by commands of this kind:

```
m = pem(Data,'P1D')
```

where `P1D` indicates one pole (time constant) and a delay. See "Process Models: the idproc Model" on page 3-41 and the reference page for `Purpose` for more details.

State-Space Models

Black-Box, Discrete Time Parameterizations

Suppose first that there is no particular knowledge about the internal structure of the discrete-time state-space model (Equation 3-15). Any linear model is sought. A simple approach is to use

```
m = pem(Data)
```

This estimates a state-space model of an order (among 1 to 10) that seems reasonable.

To find a black-box model of a certain order `n`, use

```
m = pem(Data,n)
```

To get a plot from which the order can be determined among a list of orders `nn = [n1,n2,...,nN]`, use

```
m = pem(Data,'nx',nn)
```

All these black-box models are initialized by the subspace method `n4sid`. To obtain the estimate from this routine, use

```
m = n4sid(Data,n)
```

Arbitrarily Structured Models in Discrete and Continuous Time

For state-space models of given structure, most of the effort involved relates to defining and manipulating the structure. This is discussed in "Structured State-Space Models with Free Parameters: the idss Model" on page 3-48. Once the structure is defined as `ms`, you can estimate its parameters with

```
m = pem(Data,ms)
```

When the systems are multioutput, the following criterion is used for the minimization:

$$det \sum_{t=1}^{N} e(t)e^{T}(t) \qquad \textbf{(3-42)}$$

which is the maximum likelihood criterion for Gaussian noise with unknown covariance matrix.

The numerical minimization of the prediction error criterion (Equation 3-39) or (Equation 3-42) can be a difficult problem for general model parameterizations. The criterion, as a function of the free parameters, can define a complicated surface with many local minima, narrow valleys, and so on. This can require substantial interaction from the user, in providing reasonable initial parameter values, and also by freezing certain parameter values (using the property `FixedParameters`) while allowing others to be free. Note that `pem` easily allows the freezing of any parameters to their current/nominal values. You can also directly manipulate the model structure, as described in "Structured State-Space Models with Free Parameters: the idss Model" on page 3-48. A procedure that is often used for state-space models is to allow the noise parameter in the K matrix to be free only when a reasonable model of the dynamic part has been obtained.

Optional Variables

The estimation functions accept a list of property name/property value pairs that can affect both the model structure and the estimation algorithm. For complete lists of these properties, see `algorithm properties`, `idarx`, `idmodel`, `idpoly`, `idproc`, `idss`, and `idgrey` in Chapter 4, "Functions — By Category" Some of them are listed here. Note that any property, as well as values that are strings, can be entered as any unambiguous, case-insensitive abbreviation, as in

```
m = pem(Data,mi,'fo','si')
```

Note Algorithm is a property of `idmodel`. Any algorithm property can be separately set as above. If you have a standard algorithm set up that you prefer, you can set those properties simultaneously, as in `m = pem(Data,mi,'alg',myalg)`.

Note The algorithm properties, like all other model properties, are inherited by the resulting model `m`. If you continue the estimation using `m` as the initial model, all previously set algorithm features will thus apply, unless you specify otherwise.

Applying to All Estimation Methods

The following properties apply to all estimation methods:

- `Focus`
- `MaxSize`
- `FixedParameter`

`Focus`: This property affects the weighting applied to the fit between the model and the data. It can be used to ensure that the model approximates the true system well over certain frequency intervals. `Focus` can assume the following values:

- `Prediction`: (Default) The model is determined by minimizing the prediction errors. It corresponds to a frequency weighting that is given by the input spectrum times the squared inverse noise model. Typically, this favors a good fit at high frequencies. From a statistical variance point of view, this is the optimal weighting, but then the approximation aspects (bias) of the fit are neglected.
- `Simulation`: Frequency weighting of the transfer function fit is given by the input spectrum. Frequency ranges where the input has considerable power are thus better described by the model. In other words, the model approximation is such that the model will produce as good simulations as possible when applied to inputs with the same spectra as used for the estimation. For models that have no disturbance model (A=C=D=1 for `idpoly` models and K=0 for `idss` models) there is no difference between the

simulation and prediction values. For models with a disturbance description, this is estimated by a prediction error method, keeping the estimated transfer function from input to output fixed. The resulting model is guaranteed to be stable.

- `Stability`: The algorithm is modified so that a stable model is guaranteed, but the weighting still corresponds to prediction.
- `Frequency range for passbands`: `Focus = [w1 w2]` where the interval defines a passband (in rad/s) for the signals. By letting `focus` have several rows, you can define filtering with several passbands. The model fit is then focused on the passbands defined in this way.
- `Any SISO linear filter`: The transfer function from input to output is determined by a frequency fit with this filter times the input spectrum as weighting function. The noise model is determined by a prediction error method, keeping the transfer function estimate fixed. To obtain a good model fit over a specific frequency range, the filter should thus be chosen with a passband over this range. For a model with no disturbance model, the result is the same as first applying prefiltering to data using `idfilt`. The filter can be specified as
  - Any single-input single-output `idmodel`
  - An `ss`, `tf`, or `zpk` model from the Control System Toolbox
  - `{A,B,C,D}` with the state-space matrices for the filter (notice the curly brackets)
  - `{numerator, denominator}` with the transfer function numerator/denominator of the filter

`MaxSize`: No matrix with more than `MaxSize` elements is formed by the algorithm. Instead, `for` loops are used. `MaxSize` thus decides the memory/speed tradeoff, and can prevent slow use of virtual memory. `MaxSize` can be any positive integer, but the input-output data must contain fewer than `MaxSize` elements. The default value of `MaxSize` is `Auto`, which means that the value is determined in the M-file `idmsize`. The user can edit this file to optimize speed on a particular computer. See also “Memory/Speed Tradeoffs” on page 3-98.

`FixedParameter`: A list of parameters that are kept fixed to the nominal/initial values and not estimated. This is a vector of integers containing the indices of the fixed parameters or a cell array of parameter names. If names are used, wildcard entries apply, which can be convenient if you have groups of parameters in your model. See the reference page for `Algorithm Properties`.

Algorithm Properties That Apply to n4sid, Estimating State-Space Models

The properties that apply to subspace model estimation are

- `N4Weight`
- `N4Horizon`

These properties also apply to `pem` for estimating black-box state-space models, because `pem` is initialized by the `n4sid` estimate.

`N4Weight`**:** This property determines some weighting matrices used in the singular-value decomposition that is a central step in the algorithm. Two choices are offered: `moesp`, which corresponds to the MOESP algorithm by Verhaegen, and `cva`, which is the canonical variable algorithm by Larimore. The default value is `N4Weight = Auto`, which gives an automatic choice between the two options.

`N4Horizon`**:** Determines the prediction horizons forward and backward used by the algorithm. This is a row vector with three elements: `N4Horizon =[r sy su]`, where `r` is the maximum forward prediction horizon; that is, the algorithm uses up to `r`-step-ahead predictors. `sy` is the number of past outputs, and `su` is the number of past inputs that are used for the predictions. See Ljung (1999), pages 345 to 348. These numbers can have a substantial influence on the quality of the resulting model, and there are no simple rules for choosing them. Making `N4Horizon` a `k`-by-3 matrix means that each row of `N4Horizon` is tried and the value that gives the best (prediction) fit to data is selected. If you specify only one column in `N4Horizon`, the interpretation is `r=sy=su`. The default choice is `N4Horizon = Auto`, which uses the Akaike Information Criterion (AIC) for the selection of `sy` and `su`. See the reference page for `n4sid` for literature references.

Properties That Apply to Estimation Methods Using Iterative Search for Minimizing a Criterion

The properties that govern the iterative search are

- `Trace`
- `LimitError`
- `MaxIter`
- `Tolerance`
- `SearchDirection`
- `Advanced`

These properties apply to `armax`, `bj`, `oe`, and `pem`.

`Trace`: This property determines the information about the iterative search that is provided to the MATLAB Command Window:

`Trace = Off`	No information is written to the screen.
`Trace = On`	Information about criterion values and the search process is given for each iteration.
`Trace= Full`	The current parameter values and the search direction are also given (except in the "free" `SSParameterization` case for `idss` models).

`LimitError`: This variable determines how the criterion is modified from quadratic to one that gives linear weight to large errors. Errors larger than `LimitError` times the estimated standard deviation will carry a linear weight in the criteria. The default value of `LimitError` is `1.6`. `LimitError = 0` disables the robustification and leads to a purely quadratic criterion. The standard deviation is estimated robustly as the median of the absolute deviations from the median, divided by 0.7. (See Equations (15.9) and (15.10) in Ljung (1999).)

`MaxIter`: The maximum number of iterations performed during the search for minimum. The iterations stop when `MaxIter` is reached or some other stopping criterion is satisfied. The default value of `MaxIter` is `20`. Setting `MaxIter = 0` returns the result of the startup procedure. The actual number of iterations used is given by the property `EstimationInfo.Iterations`.

`Tolerance`**:** Based on the Gauss-Newton vector computed at the current parameter value, an estimate is made of the expected improvement of the criterion at the next iteration. When this expected improvement is less than `Tolerance`%, the iterations are stopped. The default value is `0.01`.

`SearchDirection`**:** The direction along which a line search is performed to find a lower value of the criterion function. It can assume the following values:

- `gn`: The Gauss-Newton direction (inverse of the Hessian times the gradient direction). If no improvement is found along this direction, the gradient direction is also tried out.
- `gns`: A regularized version of the Gauss-Newton direction. Eigenvalues less than `pinvtol` of the Hessian are neglected, and the Gauss-Newton direction is computed in the remaining subspace. (`pinvtol` is part of the `'advanced'` field; see the `Algorithm Properties` reference page.)

- `lm`: The Levenberg-Marquardt method is used. This means that the next parameter value is `-pinv(H+d*I)*grad` from the previous one, where `H` is the Hessian, `I` is the identity matrix, and `grad` is the gradient. `d` is a number that is increased until a lower value of the criterion is found.
- `Auto`: A choice between the above is made in the algorithm. This is the default.

One property of the returned model is `EstimationInfo`, a structure that contains useful information about the estimation process. See the `EstimationInfo` reference page for a list of fields.

Another important option is `InitialState`. See "Initial State" on page 3-100.

For the spectral analysis estimate, you can compute the frequency functions at arbitrary frequencies. If the frequencies are specified in a row vector, `w`, then

```
g = spa(z,M,w)
```

results in `g` being computed at these frequencies. You can generate logarithmically spaced frequencies using the MATLAB `logspace` function. For example,

```
w = logspace(-3,pi,128)
```

Defining Model Structures

Because the System Identification Toolbox handles a wide variety of model structures, it is important that these can be defined in a flexible way. In the previous section you saw how models are automatically produced in the right form by the various estimation routines, `arx`, `iv4`, `oe`, `bj`, `armax`, and `pem`, if you just specify model orders and delays.

This section describes how model structures and models can be directly defined. This might be required, for example, when you are creating a model for simulation. It might be necessary to define model structures that are not of black-box type, but contain more detailed internal structure, reflecting some physical insights into how the system works.

The general way of representing models and model structures in the System Identification Toolbox is by various model objects. This section introduces the commands (apart from the parametric estimation functions themselves) that create these models.

The model objects will contain a number of properties. For any model you can type

```
get(m)
```

to see a list of the model's properties, and

```
set(m)
```

to see what the assignable values are. See the `get` and `set` reference pages. You can also easily retrieve each property value by subreferencing, as in

```
m.A
```

and set as in

```
m.b(3) = 27
```

See the `idmodel` reference page for complete property lists. Here only examples are given. Note that it is sufficient to use any case-insensitive, unambiguous abbreviation of the property names. Also, `'u'` is short for `'input'` and `'y'` is short for `'output'`.

Polynomial Black-Box Models: the idpoly Model

The general input-output form (Equation 3-19)

$$A(q)y(t) = \frac{B(q)}{F(q)}u(t-nk) + \frac{C(q)}{D(q)}e(t) \qquad \textbf{(3-43)}$$

is defined by the five polynomials $A(q)$, $B(q)$, $C(q)$, $D(q)$, and $F(q)$. These are represented in the standard MATLAB format for polynomials. Polynomial coefficients are stored as row vectors ordered by descending powers. For example, the polynomial

$$A(q) = 1 + a_1 q^{-1} + a_2 q^{-2} + \ldots + a_n q^{-n}$$

is represented as

```
A = [1 a1 a2 ... an]
```

Delays in the system are indicated by leading zeros in the $B(q)$ polynomial. For example, the ARX model

$$y(t) - 1.5y(t-1) + 0.7y(t-2) = 2.5u(t-2) + 0.9u(t-3) \qquad \textbf{(3-44)}$$

is represented by the polynomials

```
A = [1 -1.5 0.7]
B = [0 0 2.5 0.9]
```

The `idpoly` representation of (Equation 3-43) is now created by the command

```
m = idpoly(A,B,C,D,F,lam,T)
```

`lam` is the variance of the white noise source $e(t)$, and `T` is the sampling interval. Trailing arguments can be omitted for default values. The system (Equation 3-44) can, for example, be represented by

```
m = idpoly([1 -1.5 0.7],[0 0 2.5 0.9])
```

In the multiinput case (Equation 3-41), `B` and `F` are matrices whose row number `k` corresponds to the `k`th input. For time series (no input signal), set `B = []` and `F = []`. (See "Time-Series Modeling" on page 3-93 for more details on time series.) You can also use the command `idpoly` to define continuous-time systems. See the `idpoly` reference page for details.

When `m` is defined, the polynomials and their orders can be easily retrieved and changed, as in

```
m.a % for the A-polynomial
roots(m.a)
m.a(3)=0.95
```

Process Models: the idproc Model

A process model is a continuous-time model that is characterized by

- Static gain K_p
- One or several time constants T_{pk} (called T_w and ζ for time constant and damping in the complex case)
- A possible process zero T_z
- A possible time delay (dead time) T_d
- A possible enforced integration

This means that the models are transfer functions of the character

$$G(s) = \frac{K_p}{1 + sT_{p1}} e^{-sT_d} \qquad \textbf{(3-45)}$$

To name the different process models of interest, acronyms are used, built up from the letters

- P for process model
- 0, 1, 2 or 3, depending on the number of poles
- D when a time-delay term e^{-sT_d} is present
- Z when a process zero (numerator term) is present
- U when the poles are possibly underdamped (complex-valued poles)
- I when an integration is enforced

To illustrate this, for example,

- `P1D` for Equation 3-45
- `P2ZU` for

$$G(s) = \frac{K_p(1+sT_z)}{1+2s\zeta T_w+(sT_w)^2} \quad \textbf{(3-46)}$$

- `POID` for

$$G(s) = \frac{K_p}{s}e^{-sT_d} \quad (3\text{-}47)$$

- `P3Z` for

$$G(s) = \frac{K_p(1+sT_z)}{(1+sT_{p1})(1+sT_{p2})(1+sT_{p3})} \quad (3\text{-}48)$$

To define an `idproc` model, use the constructor

```
m = idproc('P1D')
```

where the acronym defines the character of the model. To estimate a process model from data, use

```
me = pem(data,m)
```

for an `idproc` model `m`, or more simply

```
me = pem(data,'P1D')
```

See the reference page for `idproc`. The transfer function coefficients are structures of the following kind: The parameters are called `Kp`, `Tp1`, `Tp2`, `Tp3`, `Tw`, `Zeta`, `Tz`, and `Td`, as shown above. You retrieve them by

```
Kp = get(m,'Kp') or m.Kp
```

They are structures with the following fields:

- `status`: Assumes the values `'estimate'`, `'fixed'`, or `'zero'` with obvious interpretation
- `min`: Minimum value that this parameter is bounded from below by
- `max`: Maximum value that this parameter is bounded from below by
- `value`: Numerical value of the parameter

For models with several inputs, `status` is a cell array, and `min`, `max`, and `value` are vectors of length equal to the number of inputs. Similarly, the acronym will

be a cell array indicating the characters of the transfer functions associated with the different inputs, as in `{'P1D','P2ZI'}`.

The values, status, and bounds for the parameters can be set by

```
set(m,'Kp',KC) or m.Kp = KC or m.Kp.value = 12 or m.Kp.status =
'fixed'
```

where `KC` is a structure with the correct fields. An extended syntax allows

```
m.Kp = 12 or m.Kp = 'fixed' or m.Kp = {'max',12}
```

for setting values (numerical values) and status (strings).

Similarly, at estimation time you can use

```
me = pem(data,'p1d','Kp',15)
```

to initialize the iterative search in this value, and

```
me = pem(data,'p1d','kp','fix','kp',12)
```

to fix the value of `Kp` to 12, and

```
me = pem(data,'p2z','kp',{'max',3},'kp',{'max',4})
```

to constrain the value of `Kp` to lie between 3 and 4.

Multivariable ARX Models: the idarx Model

A multivariable ARX model with *nu* inputs and *ny* outputs is given by

$$A(q)y(t) = B(q)u(t) + e(t) \quad \textbf{(3-49)}$$

Here $A(q)$ is an *ny*-by-*ny* matrix whose entries are polynomials in the delay operator q^{-1}. You can represent it as

$$A(q) = I_{ny} + A_1 q^{-1} + \ldots + A_{na} q^{-na} \quad \textbf{(3-50)}$$

as well as the matrix

$$A(q) = \begin{bmatrix} a_{11}(q) & a_{12}(q) & \dots & a_{1ny}(q) \\ a_{21}(q) & a_{22}(q) & \dots & a_{2ny}(q) \\ \dots & \dots & \dots & \dots \\ a_{ny1}(q) & a_{ny2}(q) & \dots & a_{nyny}(q) \end{bmatrix} \quad \textbf{(3-51)}$$

where the entries a_{kj} are polynomials in the delay operator q^{-1}.

$$a_{kj}(q) = \delta_{kj} + a_{kj}^{1} q^{-1} + \dots + a_{kj}^{na_{kj}} q^{-na_{kj}} \quad \textbf{(3-52)}$$

This polynomial describes how old values of output number j affect output number k. Here δ_{kj} is the Kronecker-delta; it equals 1 when $k = j$; otherwise, it is 0. Similarly, $B(q)$ is an *ny*-by-*nu* matrix

$$B(q) = B_0 + B_1 q^{-1} + \dots B_{nb} q^{-nb} \quad \textbf{(3-53)}$$

or

$$B(q) = \begin{bmatrix} b_{11}(q) & b_{12}(q) & \dots & b_{1nu}(q) \\ b_{21}(q) & b_{22}(q) & \dots & b_{2nu}(q) \\ \dots & \dots & \dots & \dots \\ b_{ny1}(q) & b_{ny2}(q) & \dots & b_{nynu}(q) \end{bmatrix} \quad \textbf{(3-54)}$$

with

$$b_{kj}(q) = b_{kj}^{1} q^{-nk_{kj}} + \dots + b_{kj}^{nb_{kj}} q^{-nk_{kj} - nb_{kj} + 1}$$

The delay from input number j to output number k is nk_{kj}. To link with the structure definition in terms of `na`, `nb`, and `nk` in the `arx` and `iv4` commands, note that `na` is a matrix whose kj element is na_{kj}, while the kj elements of `nb` and `nk` are nb_{kj} and nk_{kj}, respectively.

The `idarx` representation of the model (Equation 3-49) can be created by

```
m = idarx(A,B)
```

where `A` and `B` are 3-D arrays of dimensions `ny`-by-`ny`-by-`(na+1)` and `ny`-by-`nu`-by-`(nb+1)`, respectively, that define the matrix polynomials (Equation 3-50) and (Equation 3-53).

```
A(:,:,k+1) = Ak
B(:,:,k+1) = Bk
```

Note that `A(:,:,1)` is always the identity matrix, and that leading zero coefficients in `B` matrices define the delays.

Consider the following system with two outputs and three inputs:

$$y_1(t) - 1.5y_1(t-1) + 0.4y_2(t-1) + 0.7y_1(t-2) =$$
$$0.2u_1(t-4) + 0.3u_1(t-5) + 0.4u_2(t) - 0.1u_2(t-1) + 0.15u_2(t-2) + e_1(t)$$
$$y_2(t) - 0.2y_1(t-1) - 0.7y_2(t-2) + 0.01y_1(t-2) =$$
$$u_1(t) + 2u_2(t-4) + 3u_3(t-1) + 4u_3(t-2) + e_2(t)$$

which in matrix notation can be written as

$$y(t) + \begin{bmatrix} -1.5 & 0.4 \\ -0.2 & 0 \end{bmatrix} y(t-1) + \begin{bmatrix} 0.7 & 0 \\ 0.01 & -0.7 \end{bmatrix} y(t-2) = \begin{bmatrix} 0 & 0.4 & 0 \\ 1 & 0 & 0 \end{bmatrix} u(t) +$$
$$\begin{bmatrix} 0 & -0.1 & 0 \\ 0 & 0 & 3 \end{bmatrix} u(t-1) + \begin{bmatrix} 0 & 0.15 & 0 \\ 0 & 0 & 4 \end{bmatrix} u(t-2) + \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} u(t-3) +$$
$$\begin{bmatrix} 0.2 & 0 & 0 \\ 0 & 2 & 0 \end{bmatrix} u(t-4) + \begin{bmatrix} 0.3 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} u(t-5)$$

This system is defined and simulated for a certain input `u`, and then estimated in the correct ARX structure by the following commands:

```
A(:,:,1) = eye(2);
A(:,:,2) = [-1.5 0.4; -0.2 0];
A(:,:,3) = [0.7 0 ; 0.01 -0.7];
B(:,:,1) = [0 0.4 0;1 0 0];
B(:,:,2) = [0 -0.1 0;0 0 3];
B(:,:,3) = [0 0.15 0;0 0 4];
B(:,:,4) = [0 0 0;0 0 0];
B(:,:,5) = [0.2 0 0;0 2 0];
B(:,:,6) = [0.3 0 0;0 0 0];
m0 = idarx(A,B);
u = iddata([], idinput([200,3]));
e = iddata([], randn(200,2));
y = sim(m0, [u e]);
```

```
na = [2 1;2 2];
nb = [2 3 0;1 1 2];
nk = [4 0 0;0 4 1];
me = arx([y u],[na nb nk])
me.a % The estimated A-polynomial
```

Black-Box State-Space Models: the idss Model

The basic state-space models are the following ones. (See also "State-Space Models" on page 3-32.)

Discrete-Time Innovations Form

$$\begin{aligned} x(kT+T) &= Ax(kT)+Bu(kT)+Ke(kT) && (a)\\ y(kT) &= Cx(kT)+Du(kT)+e(kT) && (b)\\ x(0) &= x0 && (c) \end{aligned} \qquad \textbf{(3-55)}$$

Here T is the sampling interval, $u(kT)$ is the input at time instant kT, and $y(kT)$ is the output at time kT. (See Ljung (1999), page 99.)

System Dynamics Expressed in Continuous Time

$$\begin{aligned} \dot{x}(t) &= Fx(t)+Gu(t)+\tilde{K}w(t)\\ y(t) &= Hx(t)+Du(t)+w(t)\\ x(0) &= x0 \end{aligned} \qquad \textbf{(3-56)}$$

(See Ljung (1999), page 93.) It is often easier to define a parameterized state-space model in continuous time because physical laws are most often described in terms of differential equations. The matrices F, G, H, and D contain elements with physical significance (for example, material constants). The numerical values of these might or might not be known. To estimate unknown parameters based on sampled data (assuming T is the sampling interval), first transform (Equation 3-56) to (Equation 3-55) using the formulas of (Equation 3-27). The value of the Kalman gain matrix K in (Equation 3-55) or $\tilde{K}$ in (Equation 3-56) depends on the assumed character of the additive noises $w(t)$ and $e(t)$ in (Equation 3-25) and its continuous-time counterpart. Disregard that link and view K in (Equation 3-55) (or $\tilde{K}$ in (Equation 3-56)) as the basic tool to model the disturbance properties. This gives the *directly parameterized innovations form*. (See Ljung (1999), page 99.) If the internal noise structure is important, you could use user-defined gray-box structures (the `idgrey` object) as in "Parameterized Disturbance Models" on page 3-53.

You can put the discrete-time model (Equation 3-55) into the `idss` model by

```
m = idss(A,B,C,D,K,X0,'Ts',T)
```

For the continuous-time model (Equation 3-56), use

```
m = idss(F,G,H,D,Kt,X0,'Ts',0)
```

Setting the sampling interval `Ts` to zero thus means a continuous-time model. You can now use the model `m` for simulation and examine it using the various commands. The parameterization of the matrices is by default free; that is, any elements in the matrices are freely adjustable by the estimation routines. The parameters are adjusted to data by

```
me = pem(Data,m)
```

The iterative search for the best fit is then initialized in the nominal matrices `A`, `B`, `C`, `D`, `K`, `X0`. Note that the command `me = pem(Data,4)`, which just defines the model order, first estimates a starting model `m` (using `n4sid`), from which the search is initialized.

In this free parameterization, you can decide how to deal with the disturbance model matrix K. Letting

```
m.DisturbanceModel = 'None'
```

(rather than `'Estimate'`) fixes the K matrix to zero, thereby creating an output-error model.

Letting

```
m.InitialState ='zero'
```

(rather than `'Estimate'`) sets the initial state vector `x0` to zero.

The property `nk` determines the delays from the different inputs just as for `idpoly` models. Thus

```
m.nk = [0,0,...,0]
```

(no delays) means that all elements of the D matrix should be estimated, while

```
m.nk = [1,1,..,1]
```

fixes the D matrix to zero.

With the parameterization of `A`, `B`, and `C` being completely free, a basis for the state-space realization is automatically selected to give well-conditioned

calculations. An alternative is to specify an observer canonical form for A, B, C by

```
m.sspar = 'Canonical'
```

(rather than 'Free'). This is still a black-box model, because the canonical form covers all models of a certain order. The structure modifications can all be combined at the estimation call

```
me = pem(Data,m,'sspar','can','dist','none','ini','z')
```

which is the same as

```
set(m,'sspar','can','dist','none','ini','z')
me = pem(Data,m);
```

Structured State-Space Models with Free Parameters: the idss Model

The System Identification Toolbox allows you to define arbitrary parameterizations of the matrices in (Equation 3-55) or (Equation 3-56). To define the structure, so-called *structure matrices* are used. These are shadow matrices to A, B, C, D, K, and X0, have the same sizes, and coincide with these at all matrix elements that are known. The structure matrices are denoted by As, Bs, Cs, Ds, Ks, and X0s and have the entry NaN at those elements that correspond to unknown parameters to be estimated.

For example,

```
m.As = [NaN 0;0 NaN]
```

sets the structure matrix for A, called As, to a diagonal matrix, where the diagonal elements are freely adjustable. Defining

```
m.A = [2 0; 0 3]
```

sets the nominal/initial values of these diagonal elements to 2 and 3, respectively.

Example 1: A Discrete-Time Structure. Consider the discrete-time model

$$x(t+1) = \begin{bmatrix} 1 & \theta_1 \\ 0 & 1 \end{bmatrix} x(t) + \begin{bmatrix} \theta_2 \\ \theta_3 \end{bmatrix} u(t) + \begin{bmatrix} \theta_4 \\ \theta_5 \end{bmatrix} e(t)$$

$$y(t) = \begin{bmatrix} 1 & 0 \end{bmatrix} x(t) + e(t)$$

$$x(0) = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

with five unknown parameters θ_i, $i = 1,\ldots,5$. Suppose the nominal/initial values of these parameters are -1, 2, 3, 4, and 5. This structure is then defined by

```
m = idss([1, -1;0, 1],[2;3],[1,0],0,[4;5])
m.As = [1, NaN; 0 ,1];
m.Bs = [NaN;NaN];
m.Cs = [1, 0];
m.Ds = 0;
m.Ks = [NaN;NaN];
m.x0s = [0;0];
```

The definition thus follows in two steps. First the nominal model is defined. Then the structure (known and unknown parameter values) is defined by the structure matrices `As`, `Bs`, etc. The command `setstruc` makes the above syntax more efficient.

Example 2: A Continuous-Time Model Structure. Consider the following model structure:

$$\dot{x}(t) = \begin{bmatrix} 0 & 1 \\ 0 & \theta_1 \end{bmatrix} x(t) + \begin{bmatrix} 0 \\ \theta_2 \end{bmatrix} u(t)$$

$$y(t) = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} x(t) + e(t)$$

$$x(0) = \begin{bmatrix} \theta_3 \\ 0 \end{bmatrix}$$

This corresponds to an electrical motor, where $y_1(t) = x_1(t)$ is the angular position of the motor shaft and $y_2(t) = x_2(t)$ is the angular velocity. The

parameter $-\theta_1$ is the inverse time constant of the motor and $-\theta_2/\theta_1$ is the static gain from the input to the angular velocity. (See Example 4.1 in Ljung (1999).) The motor is at rest at time 0 but at an unknown angular position. Suppose that θ_1 is around -1 and θ_2 is around 0.25. If you also know that the variance of the errors in the position measurement is 0.01 and in the angular velocity measurements is 0.1, you can then define an `idss` model using

```
m = idss([0 1;0   ...
    -1],[0;0.25],eye(2),[0;0],zeros(2,2),[0;0],'Ts',0)
m.as = [0 1; 0 NaN]
m.bs = [0 ;NaN]
m.cs = m.c
m.ds = m.d
m.ks = m.k
m.x0s = [NaN;0]
m.noisevar = [0.01 0; 0 0.1]
```

You can now use the structure `m` to estimate the unknown parameters θ_i from observed data

```
Data = iddata([y1 y2], u, 0.1)
```

by

```
model = pem(Data,m)
```

The iterative search for minimum is then initialized at the parameters in the nominal model `m`. The continuous-time model is automatically sampled to agree with the sampling interval of the data. You can also use the structure to simulate the system above with sampling interval `T = 0.1` for input `u` and noise realization `e`.

```
e = randn(300,2)
u = idinput(300);
simdat = iddata([],[u e],'Ts',0.1);
y = sim(m,simdat) % The continuous system will automatically be
                  % sampled using Ts = 0.1.
```

The nominal parameter values are used, and the noise sequence is scaled according to the matrix `m.noisevar`.

When estimating models, you can try a number of neighboring structures, such as “What happens if I fix this parameter to a certain value?” or “What happens if I free these parameters?” This is easily handled by the structure matrices `As`,

Bs, etc. For example, to free the parameter $x_2(0)$ (perhaps the motor wasn't at rest after all), you can use

```
model = pem(Data,m,'x0s',[NaN;NaN])
```

To manipulate initial conditions, the function `init` is also useful.

State-Space Models with Coupled Parameters: the idgrey Model

In some situations you might want the unknown parameters in the matrices in (Equation 3-55) or (Equation 3-56) to be linked to each other. Then the `NaN` feature is not sufficient to describe these links. Instead you need to do some gray-box modeling and write an M-file that describes the structure. The format is

```
[A,B,C,D,K,x0] = mymfile(par,T,aux);
```

where `mymfile` is the user-defined name for the M-file, `par` contains the parameters as a column vector, `T` is the sampling interval, and `aux` contains auxiliary variables. The latter variables are used to house options, so that you can try out some different cases without your having to edit the M-file. The matrices `A`, `B`, `C`, `D`, `K`, and `x0` refer either to the continuous-time description (Equation 3-56) or to the discrete-time description (Equation 3-55). When a continuous-time description is fitted to sampled data, the estimation routines perform the necessary sampling of the model. To obtain the same structure as in "Example 2: A Continuous-Time Model Structure" you can do the following:

```
function [A,B,C,D,K,x0] = mymfile(par,T,aux)
A = [0 1; 0 par(1)];
B = [0;par(2)];
C = eye(2);
D = zeros(2,2);
K = zeros(2,1);
x0 =[par(3);0];
```

Once you have written the M-file, the `idgrey` model `m` is defined by

```
m = idgrey('mymfile',par,'c',aux);
```

where `par` contains the nominal (initial) values of the corresponding entries in the structure. `'c'` signals that the underlying parameterization is continuous

time. aux contains the values of the auxiliary parameters. Note that T and aux must be given as input arguments, even if they are not used by the code.

From here on, estimate models and evaluate results as for any other model structure. Some further examples of user-defined model structures are given below.

Some Examples of idgrey Model Structures

With user-defined structures, you have complete freedom in the choice of models of linear systems. This section gives two examples of such structures.

Heat Diffusion. Consider a system driven by the heat-diffusion equation (see also Example 4.3 in Ljung (1999)).

This is a metal rod with a heat-diffusion coefficient κ, which is insulated at the near end and heated by the power u (W) at the far end. The output of the system is the temperature at the near end. This system is described by a partial differential equation in time and space. Replacing the space-second derivative by a corresponding difference approximation gives a continuous-time state-space model (Equation 3-56), where the dimension of x depends on the grid size in space used in the approximation. It is also desirable to be able to work with different grid sizes without having to edit the model file. This is described by the following M-file:

```
function [A,B,C,D,K,x0] = heatd(pars,T,aux)
Ngrid = aux(1); % Number of points in the space-discretization
L = aux(2); % Length of the rod
temp = aux(3); % Assuming uniform initial temperature of the rod
deltaL = L/Ngrid;    % Space interval
kappa = pars(1); % The heat-diffusion coefficient
htf = pars(2); % Heat transfer coefficient at far end of rod
A = zeros(Ngrid,Ngrid);
for kk = 2:Ngrid-1
A(kk,kk-1) = 1;
A(kk,kk) = -2;
A(kk,kk+1) = 1;
end
A(1,1) = -1; A(1,2) = 1; % Near end of rod insulated
A(Ngrid,Ngrid-1) = 1;
A(Ngrid,Ngrid) = -1;
A = A*kappa/deltaL/deltaL;
```

```
B = zeros(Ngrid,1);
B(Ngrid,1) = htf/deltaL;
C = zeros(1,Ngrid);
C(1,1) = 1;
D = 0;
K = zeros(Ngrid,1);
x0 = temp*ones(Ngrid,1);
```

You can then define the model by

```
m = idgrey('heatd',[0.27 1],'c',[10,1,22])
```

for a tenth-order approximation of a heat rod one meter in length with an initial temperature of 22 degrees. The initial estimate of the heat conductivity is 0.27, and of the heat transfer coefficient is 1.

The model parameters are estimated by

```
me = pem(Data,m)
```

If you would like to try a finer grid, that is, take `Ngrid` larger, you can do this easily with

```
me = pem(Data,m,'Filearg',[20,1,22])
```

Parameterized Disturbance Models. Consider a discrete-time model

$$x(t+1) = Ax(t) + Bu(t) + w(t)$$
$$y(t) = Cx(t) + e(t)$$

where w and e are independent white noises with covariance matrices *R1* and *R2*, respectively. Suppose that you know the variance of the measurement noise *R2*, and that only the first component of $w(t)$ is nonzero. This can be handled by the following M-file:

```
function [A,B,C,D,K,x0] = mynoise(par,T,aux)
R2 = aux(1); % The assumed known measurement noise variance
A = [par(1) par(2);1 0];
B = [1;0];
C = [par(3) par(4)];
D = 0;
R1 = [par(5) 0;0 0];
K = A*dlqe(A,eye(2),C,R1,R2); % From the Control System Toolbox
x0 = [0;0];
```

State-Space Structures: Initial Values and Numerical Derivatives

For a structured state-space model, it is sometimes difficult to find good initial parameter values at which to start the numerical search for a minimum of (Equation 3-38). It is always best to use physical insight, whenever possible, to suggest such values. For random initialization, the command `init` is useful. Because there is always a risk that the numerical minimization can get stuck in a local minimum, it is advisable to try several different initialization values for θ.

In the search for the minimum, the gradient of the prediction errors $e(t)$ with respect to the parameters is computed by numerical differentiation. The step size is determined by the M-file `nuderst`. In its default version, the step size is simply 10^{-4} times the absolute value of the parameter in question (or the number 10^{-7} if this is larger). When the model structure contains parameters with different orders of magnitude, try to scale the variables so that the parameters are all roughly the same magnitude. You might need to edit the M-file `nuderst` to address the problem of suitable step sizes for numerical differentiation.

Estimating Continuous-Time Models: General Remarks

In many cases you want to estimate a continuous-time model. The System Identification Toolbox gives several ways for you to do this:

- Estimate a discrete-time model, and convert it to continuous time, using the command `d2c`. Note that the estimated model contains information about the input intersample properties of the estimation data, and the conversion, by default, will be in accordance with this information. If a polynomial type model (`idpoly` model) is estimated, you can choose the number of numerator and denominator coefficients freely for the discrete-time model. Note, however, that the transformed continuous-time model generically has a numerator order that is one less than (or equal to, if `nk = 0`) the denominator order, regardless of the discrete-time orders.

  ```
  m = oe(data,[3 4 1]);
  mc = d2c(m)
  ```

- Use continuous-time frequency-domain data and directly estimate an `idpoly` continuous-time model. Then you can choose the numerator and

denominator orders freely. In the case below it is assumed that the data is sampled so fast (or that the input is band limited) that frequencies above the Nyquist frequency in the continuous-time input are negligible.

```
DF = fft(data)
DF.ts = 0 (% treating the frequency data as continuous time)
m = oe(DF,[nb nf])
```

Here `nb` is the number of numerator coefficients and `nf` the number of denominator coefficients. The delay order, `nk`, has no meaning for continuous-time OE models, and should be omitted. This means that for `nb = 2`, `nf = 4` the model is

$$G(s) = \frac{b_1 s + b_2}{s^4 + f_1 s^3 + f_2 s^2 + f_3 s + f_4} \qquad \textbf{(3-57)}$$

If the data is sampled fast, it is usually a good idea to apply some lowpass filtering before making the fit. This is most easily done with the `focus` property.

```
m = oe(DF,[nb nf],'focus',[0 10])
```

meaning that only data in the frequency interval between 0 and 10 rad/s is used in the model estimation.

Of course, you can also use continuous-time frequency-domain data to estimate continuous-time state-space models.

```
m = pem(DF,n)
```

This gives an `n`th-order continuous-time state-space model with no direct term (`D` matrix = 0). To include a `D` matrix, indicate that the relative degree `nk` is zero.

```
m = pem(DF,n,'nk',0)
```

- Use continuous-time process models as described in “Process Models: the idproc Model” on page 3-41.

  ```
  m = pem(data,'P1D')
  ```

- Build a continuous-time `idgrey` model as described in “State-Space Models with Coupled Parameters: the idgrey Model” on page 3-51, and in Example 3.3 on page 3-52.

- Build a continuous-time `idss` model either in structured or in free form as described in "Black-Box State-Space Models: the idss Model" on page 3-46.

  In the latter three cases you can directly estimate the continuous-time model from discrete-time data (or continuous-time frequency-domain data) without further information, because the `iddata` object contains all relevant information to adjust the model to the data.

  ```
  m = pem(Data,mi)
  ```

  If you create a state-space model at the same time as you estimate it, you must, however, indicate whether you want to obtain a continuous-time model.

  ```
  m = n4sid(Data,5,'Ts',0)
  m = pem(Data,5,'Ts',0,'ss','can')
  ```

  In the `pem` example, the `('ss','can')` property name/property value pair means that the state-space parameterization (`'SSparameterization'`) is `'Canonical'`. Estimation of freely parameterized continuous-time state-space models is not supported.

Examining Models

Once you have estimated a model, you need to investigate its properties. You have to simulate it, test its predictions, and compute its poles and zeros and so on. You thus have to transform the model to various ways of representing and presenting it. This section deals with how this is done. The following topics are covered:

- Parametric models: basic use, accessing properties, simulation, and prediction. Also manipulating channels, in particular the noise input channels.
- Frequency-domain models
- Graphing model properties
- Transformations to other representations
- Transformations between continuous and discrete time

Parametric Models: idmodel and Its Children

`idmodel` is an object that you do not deal with directly. It contains all the common properties of the model objects `idarx`, `idgrey`, `idpoly`, `idproc`, and `idss`, which are returned by the different estimation routines.

Basic Use

If you just estimate models from data, the model objects should be transparent. All parametric estimation routines return `idmodel` results.

```
m = arx(Data,[2 2 1])
```

The model `m` contains all relevant information. Just typing `m` gives a brief account of the model. `present(m)` also gives information about the uncertainties of the estimated parameters. `get(m)` gives a complete list of model properties.

Most of the interesting properties can be directly accessed by subreferencing.

```
m.a
m.da
```

See the property list obtained by `get(m)`, as well as the property lists of `idgrey`, `idarx`, `idpoly`, `idproc`, and `idss`, in Chapter 4, “Functions — By Category” for more details on this.

You can directly examine and display the characteristics of the model `m` by using commands like `impulse`, `step`, `bode`, `nyquist`, and `pzmap`. Use commands like `compare` and `resid` to assess the quality of the model. If you have the Control System Toolbox, typing `view(m)` gives you access to various display functions. More details about this are given below.

To extract state-space matrices, transfer function polynomials, etc., you can use these commands:

```
arxdata, polydata, tfdata, ssdata, zpkdata
```

To compute the frequency response of the model, you can use `idfrd` and `freqresp`.

Simulation and Prediction

Any `idmodel` `m` can be simulated with

```
y = sim(m,Data)
```

where `Data` is an `iddata` object with just input channels.

```
Data = iddata([ ],[u v])
```

The number of input channels must either be equal to the number of measured input channels in `m`, in which case a noise-free simulation is obtained, or equal to the sum of the number of input and output channels in `m`. In the latter case the last input signals (`v`) are interpreted as white noise. They are then scaled by the `NoiseVariance` matrix of `m` and added to the output via the disturbance model

$$y = Gu + He$$
$$e = Lv$$

where the matrix L is given from the noise covariance Λ by $\Lambda = LL^T$.

```
L=chol(m.NoiseVariance)'
```

The output is returned as an `iddata` object with just output channels. Here is a typical string of commands.

```
A = [1 -1.5 0.7];
B = [0 1 0.5];
m0 = idpoly(A,B,[1 -1 0.2]);
u = iddata([],idinput(400,'rbs',[0 0.3]));
```

```
v= iddata([],randn(400,1));
y = sim(m0, [u v]);
plot(y)
```

The inverse model (Equation 3-38), which computes the prediction errors from given input-output data, is simulated with

```
e = pe(m,[y u])
```

To compute the `k`-step-ahead prediction of the output signal based on a model `m`, the procedure is as follows:

```
yhat = predict(m,[y u],k)
```

The predicted value $\hat{y}(t|t-k)$ is computed using the information in $u(s)$ up to time $s = t$ and information in $y(s)$ up to time $s = t-k$. The actual way that the information in past outputs is used depends on the disturbance model in `m`. For example, an output-error model (that is, $H = 1$ in (Equation 3-10)) maintains that there is no information in past outputs; therefore, predictions and simulations coincide.

`predict` can evaluate how well a time-series model is capable of predicting future values of the data. In this example `y` is the original series of monthly sales figures. A model is estimated based on the first half, and then its ability to predict half a year ahead is tested on the second half of the observations.

```
plot(y)
y1 = y(1:48), y2 = y(49:96)
m4 = ar(y1,4)
yhat = predict(m4,y2,6)
plot(y2,yhat)
```

The command `compare` is useful for any comparisons involving `sim` and `predict`.

Dealing with Input and Output Channels

For multivariable models, you construct submodels each containing a subset of inputs and outputs by simple subreferencing. The outputs and input channels can be referenced according to

```
m(outputs,inputs)
```

Use the colon (:) to denote all channels and the empty matrix ([ ]) to denote no channels. The channels can be referenced by number or by name. For several names, you must use a cell array.

```
m3 = m('position',{'power','speed'})
```

or

```
m3 = m(3,[1 4])
```

Thus `m3` is the model obtained from `m` by considering the transfer functions from input numbers `1` and `4` (with input names `'power'` and `'speed'`) to output number `3` (with name `'position'`).

For a single-output model `m`,

```
m4 = m(inputs)
```

selects the corresponding input channels, and for a single-input model

```
m5 = m(outputs)
```

selects the indicated output channels.

Subreferencing is quite useful, for example, when you want a plot of just some channels.

Noise Channels

The estimated models have two kinds of input channels: the measured inputs u and the noise inputs e. For a general linear model `m`,

$$y(t) = G(q)u(t) + H(q)e(t) \qquad \textbf{(3-58)}$$

where `u` is the nu-dimensional vector of measured input channels and `e` is the ny-dimensional vector of noise channels. The covariance matrix of `e` is given by the property `'NoiseVariance'`. Occasionally this matrix Λ is written in factored form:

$$\Lambda = LL^T$$

This means that `e` can be written as

$$e = Lv$$

where `v` is white noise with identity covariance matrix (independent noise sources with unit variances).

If `m` is a time series (*nu* `= 0`), *G* is empty and the model is given by

$$y(t) = H(q)e(t) \qquad \textbf{(3-59)}$$

For the model `m` in (Equation 3-58), the restriction to the transfer function matrix *G* is obtained by

`m1 = m('measured')` or just `m1 = m('m')`

Then `e` is set to `0` and `H` is removed.

Analogously

`m2 = m('noise')` or just `m2 = m('n')`

creates a time-series model `m2` from `m` by ignoring the measured input. `m2` is given by (Equation 3-59).

For a system with measured inputs, `bode`, `step`, and many other transformation and display functions just deal with the transfer function matrix *G*. To obtain or graph the properties of the disturbance model *H*, it is therefore important to make the transformations `m('n')`. For example,

```
bode(m('n'))
```

will plot the additive noise spectra according to the model `m`, while

```
bode(m)
```

just plots the frequency responses of *G*.

To study the noise contributions in more detail, it might be useful to convert the noise channels to measured channels, using the command `noisecnv`:

```
m3 = noisecnv(m)
```

This creates a model `m3` with all input channels, both measured `u` and noise sources `e`, being treated as measured signals. That is, `m3` is a model from `u` and `e` to `y`, describing the transfer functions *G* and *H*. The information about the variance of the innovations `e` is then lost. For example, studying the step response from the noise channels does not take into consideration how large the noise contributions actually are.

To include that information, you should normalize `e` first, $e = Lv$, so that v becomes white noise with an identity covariance matrix.

```
m4 = noisecnv(m,'Norm')
```

This creates a model `m4` with u and v treated as measured signals.

$$y(t) = G(q)u(t) + H(q)Lv(t) = \begin{bmatrix} G & HL \end{bmatrix} \begin{bmatrix} u \\ v \end{bmatrix}$$

For example, the step responses from v to y will now also reflect the typical size of the disturbance influence because of the scaling by L. In both these cases, the previous noise sources that have become regular inputs will automatically get input names that are related to the corresponding output. The unnormalized noise sources `e` have names like `'e@y1'` (noise `e` at output channel with name `y1`), while the normalized sources `v` are called `'v@y1'`.

Retrieving Transfer Functions

The functions that retrieve transfer function properties, `ssdata`, `tfdata`, and `zpkdata`, will thus work as follows for a model (Equation 3-58) with measured inputs. (*`fcn`* is any of `ssdata`, `tfdata`, or `zpkdata`.)

- *`fcn`*`(m)` returns the properties of G (ny outputs and nu inputs).
- *`fcn`*`(m('n'))` returns the properties of the transfer function H (ny outputs and ny inputs).
- *`fcn`*`(noisecnv(m))` returns the properties of the transfer function $[G\ H]$ (ny outputs and ny+nu inputs).
- *`fcn`*`(noisecnv(m,'Norm'))` returns the properties of the transfer function $[G\ HL\}$ (ny outputs and ny+nu inputs). Analogously,

  ```
  fcn(noisecnv(m('n'),'Norm'))
  ```

  returns the properties of the transfer function HL (ny outputs and ny inputs).
- If `m` is a time-series model, *`fcn`*`(m)` returns the properties of H, while

  ```
  fcn(noisecnv(m,'Norm'))
  ```

  returns the properties of HL.

Note that the estimated covariance matrix `NoiseVariance` itself is uncertain. This means that the uncertainty information about H is different from that of HL.

idmodel Properties

See the `idmodel` reference page for a complete list of `idmodel` properties.

Adding Channels

```
m = [m1,m2,...,mN]
```

creates an `idmodel` object `m`, consisting of all the input channels in `m1,... mN`. The output channels of `mk` must be the same. Analogously,

```
m = [m1;m2;... ;mN]
```

creates an `idmodel` object `m` consisting of all the output channels in `m1`, `m2,...`, `mN`. The input channels of `mk` must all be the same.

If you have the Control System Toolbox, you can create interconnections between `idmodels`, like `G1+G2`, `G1*G2`, `append(G1,G2)`, `feedback(G1,G2)`, etc., just as for LTI objects. However, covariance information is typically lost.

Frequency Function Format: the idfrd Model

Frequency functions and spectra are stored as an `idfrd` (Identified Frequency Response Data) model object (which is not a child of `idmodel`). This model format is used by `spa`,`spafdr`, and `etfe` to deliver their results. Moreover, any `idmodel` can be transformed to an `idfrd` object.

The frequency function and the disturbance spectrum corresponding to an `idmodel` `m` are computed by

```
h = idfrd(m)
```

This gives G and $\hat{\Phi}_v$ in (Equation 3-11) along with their estimated covariances, which are translated from the covariance matrix of the estimated parameters. The frequencies can be specified as in `h = idfrd(m,w)`, but otherwise a default choice of frequencies (based on the dynamics of `m`) is used. If `m` corresponds to a continuous-time model, the frequency functions are computed accordingly.

You retrieve the functions using `h.ResponseData`, `h.CovarianceData`, `h.SpectrumData`, and `h.NoiseCovariance`, or any case-insensitive abbreviation of the names. The frequency vector is contained in `h.Frequency`.

In addition, you can define an `idfrd` model directly from the frequency functions. See the `idfrd` reference page, which also contains a list of `idfrd` properties. The channels of an `idfrd` model can be manipulated analogously to `idmodel`s.

An alternative is to compute the response functions without storing them as `idfrd` objects:

```
[Response,Frequency,Covariance] = freqresp(m)
```

Graphs of Model Properties

There are several commands in the toolbox for graphing model characteristics.

- `bode`
- `compare`
- `ffplot`
- `impulse`
- `nyquist`
- `pzmap`
- `step`

They all have the same basic syntax. To look at one model, use

```
command(Model)
```

where `command` is any of the functions listed above.

```
command(Mod1,Mod2,...,ModN)
```

shows a comparison of several models. `Modk` can be any `idmodel` models. They can be used with any of the Control System Toolbox's LTI models. For some commands `Modk` can also be `idfrd` and `iddata` objects. For multivariable models, the plots are grouped so that each input/output channel (for all models) is plotted together. The `InputName` and `OutputName` properties of the models are used for this. The number of channels need not be the same in the different models, which is quite useful when you are trying to find a good model of a multivariable system.

```
command(Mod1,PlotStyle1,...,ModN,PlotStyleN)
```

allows you to define colors, line styles, and markers associated with the different models. `PlotStyle` takes values such as `'b'` (for blue), `'b:'` (for a blue dotted line), or `'b*-'` (for a blue solid line with the points marked by a star). This is the same as for the usual `plot` command.

To show the uncertainty of the model characteristics, use

```
command(Mod1,...,ModN,'sd',SD)
```

The plot will show dash-dotted lines that mark a confidence region around the nominal model corresponding to `SD` standard deviations (for the Gaussian distribution). This region is computed using the estimated covariance matrix for the estimated parameters.

```
command(Mod1,...,ModN,'sd',SD,'fill')
```

shows the uncertainty region as a filled region instead.

The commands have some further options to select time or frequency ranges. See the detailed descriptions in Chapter 4, “Functions — By Category”

If `Model` contains measured input channels, the plot shows just the transfer functions from these measured inputs to the outputs, that is, G in (Equation 3-58). To graph the response from the noise sources, use

```
command(Model('n'))
```

For the frequency-response graphs, this shows the additive disturbance spectra, that is, the spectra of the signal $H(q)e(t)$ in Equation 3-58, so that the properties of the noise source e are included in the plot.

For the other graphs, the properties of the transfer function H are shown. That is, no noise normalization is done. The same is true if `Model` is a time series and has no measured input channels. That means that, for example, `step` shows the step response of the transfer function H, without accounting for the size (covariance matrix) of e. To include such effects, the disturbances should first be converted to normalized noise sources, using the command `noisecnv`. See “Noise Channels” on page 3-60.

Model Output

An important and visually suggestive plot is to compare the measured output signal with the models’ simulated or predicted outputs. You do this using

```
compare(Data,model)
```

The input signal in `Data` is used by the models to simulate the output. This simulated output is shown together with the measured output, which reveals what features in the data the model can and cannot reproduce. A legend shows the fit between the signals, in terms of how much of the output variation is reproduced by the models.

Frequency Response

Three functions offer graphic display of the frequency functions and spectra: `bode`, `ffplot`, and `nyquist`.

```
bode(G)
```

plots the Bode diagram of `G` (logarithmic scales and frequencies in rad/s). If `G` is a spectrum, only an amplitude plot (the power spectrum) is given. Here `G` can be any `idmodel` or `idfrd` object.

The command `ffplot` has the same syntax as `bode` but works with linear frequency scales and Hertz as the unit. The command `nyquist` also has the same syntax, but produces Nyquist plots, that is, graphs of the frequency function in the complex plane.

Transient Response

The impulse and step responses of the models are shown by

```
impulse(Model)
```

and

```
step(Model)
```

`impulse` and `step` follow the general syntax, but can also accept `iddata` objects as arguments. For direct estimation of step and impulse responses from data, use the procedure described in “Estimating Impulse Responses” on page 3-15.

Zeros and Poles

The zeros and poles are graphed by

```
pzmap(Model)
```

This gives a plot with `'x'` marking poles and `'o'` marking zeros. Otherwise, `pzmap` follows the general syntax.

General

If you have the Control System Toolbox,

```
view(Model)
```

opens the LTI viewer with access to a number of model displays. No uncertainty information can be shown, however.

Transformations to Other Model Representations

Within the structure in which the model was created, you can extract parametric information using the `get` function or by subscripting. For example, for a state-space model, `Mod.A` is the A matrix, while `Mod.dA` contains its standard deviations. For a polynomial model, `Mod.a` and `Mod.da` are the A polynomial and its standard deviation.

Generally speaking you can transform to another representation by just using the object constructor, as in

```
modss = idss(Model)
modp = idpoly(Model)
```

Analogously, if you have the Control System Toolbox, you can freely transform between the different `idmodel` objects and the `LTI` objects.

```
syss = ss(Model)
systf = tf(Model)
Model = idss(Ltisys)
```

In addition, regardless of the particular model structure, there are a number of commands that compute various model representations. These all have the basic syntax

```
[G, dG] = command(Model)
```

where `G` contains model characteristics and `dG` their standard deviation or covariance. The transformation commands are

```
[A,B,C,D,K,X0,dA,dB,dC,dD,dK,dX0] = ssdata(Model)

[a,b,c,d,f,da,db,dc,dd,df] = polydata(Model)

[A,B,dA,dB] =arxdata(Model)

[Num,Den,dNum,dDen] = tfdata(Model)
```

```
[Z,P,K,CovZ,CovP,covK] = zpkdata(Model)

G = idfrd(Model)

[H,w,CovH] = freqresp(Model)
```

The two last commands were described previously. The three first commands clearly transform to the state-space, the polynomial, and the multivariable ARX representations. See "Defining Model Structures" on page 3-39. `tfdata` and `zpkdata` compute the transfer functions and zeros, poles, and transfer function gains. See Chapter 4, "Functions — By Category" for details.

Discrete- and Continuous-Time Models

Continuous-Time Models

Continuous-time models are created and recognized by the property `'Ts' = 0`. You can create and analyze all `idmodel` objects as continuous-time models by setting `Ts` equal to zero at the time of creation, as in

```
m = idpoly(1,[0 1 1],1,1,[1 2 3],'Ts',0)
```

for the model

$$y = \frac{s+1}{s^2+2s+3}u + e$$

All model characteristics are then computed and graphed for the continuous-time representation. Time and frequency scales are determined based on the dynamics of the system (the pole/zero locations).

For simulation and prediction, the continuous-time models are first converted to discrete time, using the sampling interval and intersample behavior of the data.

Estimating Continuous-Time Models

The estimation routines support the estimation of continuous-time state-space models in several different ways. This was described in "Estimating Continuous-Time Models: General Remarks" on page 3-54.

The major reason for identifying continuous-time models is to secure a particular structure of the continuous-time state-space matrices. This would typically reflect a physical interpretation or some gray-box modeling work

done, as for the process models, described in "Process Models: the idproc Model" on page 3-41, or continuous-time `idss` or `idgrey` models, as described in "Black-Box State-Space Models: the idss Model" on page 3-46.

Transformations

Transformations between continuous-time and discrete-time model representations are performed by `c2d` and `d2c`. Note that it is not sufficient just to assign a new value of `Ts` to the model object. The corresponding uncertainty measure (the estimated covariance matrix of the internal parameters) is also transformed in most cases. The syntax is

```
modc = d2c(modd)
modd = c2d(mc,T)
```

The transformation `c2d` also offers an optional output argument that describes how the initial state should be transformed.

If the discrete-time model has some pure time delays ($nk > 1$), the default command removes them before forming the continuous-time model, and appends them using the property `InputDelay` in model `modc`. This property is used to add appropriate phase lag and shift the data whenever the model is used. `d2c` also offers an option to approximate the dead time by a finite dimensional system. Note that the disturbance properties are translated by the somewhat questionable formula (Equation 3-29). The covariance matrix is translated by the Gauss approximation formula using numerical derivatives. The M-file `nuderst` is then invoked. You might want to edit it for applications where the parameters have very different orders of magnitude. See the comments in "State-Space Structures: Initial Values and Numerical Derivatives" on page 3-54.

Here is an example that compares the Bode plots of an estimated model and its continuous-time counterpart.

```
m= armax(Data,[2 3 1 2]);
mc = d2c(m); bode(m,mc)
```

The transformations between discrete and continuous time depend on the intersample behavior of the input. The formulas are different if the input is assumed to be piecewise constant or piecewise linear between samples (`'zoh'` or `'foh'`). For estimated discrete-time models, the input properties of the estimation data are used for this purpose, by default. To override this, add an extra argument, as described in the reference pages for `c2d` and `d2c`.

Model Structure Selection and Validation

After you have been analyzing data for some time, you typically end up with a large collection of models with different orders and structures. You need to decide which one is best, and whether the best description is an adequate model for your purposes. These are the problems of *model validation*.

Model validation is the heart of the identification problem, but there is no absolute procedure for approaching it. It is wise to be equipped with a variety of different tools with which to evaluate model qualities. The command `advice` can be applied to any estimated model for some hints on the model's quality.

```
advice(Model)
```

This section describes the techniques you can use to evaluate model qualities using the System Identification Toolbox.

Comparing Different Structures

It is natural to compare the results obtained from model structures with different orders. For state-space models, you can easily obtain this by using a vector argument for the order in `n4sid` or `pem`.

```
m = n4sid(Data,1:10)
m = pem(Data,'nx',3:15)
```

This invokes a plot from which a best order is chosen. If you omit the order argument, `m = n4sid(Data)` or `pem(Data)` makes a default choice of the best order.

For models of ARX type, various orders and delays can be efficiently studied with the command `arxstruc`. Collect in a matrix `NN` all the ARX structures you want to investigate, so that each row of `NN` is of the type

```
[na nb nk]
```

With

```
V = arxstruc(Date,Datv,NN)
```

an ARX model is fitted to the data set `Date` for each of the structures in `NN`. Next, for each of these models, the sum of squared prediction errors is computed as they are applied to the data set `Datv`. The resulting loss functions are stored in `V` together with the corresponding structures.

To select the structure that has the smallest loss function for the validation set `Datv`, use

```
nn = selstruc(V,0)
```

Such a procedure is known as *cross validation* and is a good way to approach the model selection problem.

It is usually a good idea to visually inspect how the fit changes with the number of estimated parameters. You can get a graph of the fit versus the number of parameters with

```
selstruc(V)
```

This routine prompts you to choose the number of parameters to estimate, based upon visual inspection of the graph. Then it selects the structure with the best fit for that number of parameters.

The command `struc` helps generate typical structure matrices `NN` for single-input systems. A typical sequence of commands is

```
V = arxstruc(Date,Datv,struc(2,2,1:10));
nn = selstruc(V,0);
nk = nn(3);
V = arxstruc(Date,Datv,struc(1:5,1:5,nk-1:nk+1));
selstruc(V)
```

where you first establish a suitable value of the delay `nk` by testing second-order models with delays between 1 and 10. The best fit selects the delay, and then all combinations of ARX models with up to five a and b parameters are tested with delays around the chosen value (a total of 75 models).

If the model is validated on the same data set from which it was estimated, that is, if `Date = Datv`, the fit always improves as the flexibility of the model structure increases. You need to compensate for this automatic decrease of the loss functions. There are several approaches for this. Probably the best known technique is Akaike's Final Prediction Error (FPE) criterion and his closely related Information Theoretic Criterion (AIC). Both simulate the cross-validation situation, where the model is tested on another data set.

The FPE is formed as

$$FPE = \frac{1 + \frac{d}{N}}{1 - \frac{d}{N}} V$$

where d is the total number of estimated parameters and N is the length of the data record. `V` is the loss function (quadratic fit) for the structure in question. The AIC is formed as

$$AIC = \log\left(V\left(1 + 2\frac{d}{N}\right)\right)$$

(See Section 16.4 in Ljung (1999).)

According to Akaike's theory, in a collection of different models, choose the one with the smallest FPE (or AIC). You can display the FPE values with the model parameters by typing just the model name. It is also one of the fields in `EstimationInfo`, and you can access it using

```
FPE = fpe(m)
```

Similarly, the AIC value of an estimated model is obtained as

```
AIC = aic(m)
```

If you have used `arxstruc` to generate many ARX models, you find the structure that minimizes the AIC by

```
nn = selstruc(V,'AIC')
```

where `V` is the output of `arxstruc`. A related criterion is Rissanen's Minimum Description Length (MDL) approach, which selects the structure that allows the shortest overall description of the observed data. This is obtained with

```
nn = selstruc(V,'MDL')
```

If substantial noise is present, the ARX models might need to be of high order to describe simultaneously the noise characteristics and the system dynamics. (For ARX models the disturbance model $1/A(q)$ is directly coupled to the dynamics model $B(q)/A(q)$.)

Impulse Response to Determine Delays

The command `impulse` applied to a data set

```
impulse(Data,'sd',3)
```

shows a nonparametric estimate of the impulse response. In the call above, a confidence region around zero is also shown, corresponding to three standard deviations (ca. 99.9%). Any part of the impulse response that is outside this region is thus significant. The first sample after $t = 0$, at which the impulse response estimate crosses the confidence band, is thus a good estimate of the delay in the channel in question.

Significant impulse response estimates for negative time lags are indications of feedback in the data.

Checking Pole-Zero Cancellations

A near pole-zero cancellation in the dynamics model is an indication that the model order might be too high. To judge whether a near cancellation is a real cancellation, take the uncertainties in the pole and zero locations into consideration

```
pzmap(mod,'sd',1)
```

where the `1` indicates how many standard deviations wide the confidence interval is. If the confidence regions of a zero and a pole overlap, try lower model orders.

This check is especially useful when the models have been generated by `arx`. As mentioned previously, the orders can be pushed up because of the requirement that $C/A(q)$ describe the disturbance characteristics. Checking cancellations in $B(q)/A(q)$ then gives a good indication of which orders to choose from model structures like `armax`, `oe`, and `bj`.

Residual Analysis

The residuals associated with the data and a given model, as in (Equation 3-38), are ideally white and independent of the input for the model to correctly describe the system. The function

```
resid(Model,Data)
```

computes the residuals (prediction errors) e from the model when applied to `Data`, and performs whiteness and independence analyses. The autocorrelation

function of e and the cross-correlation function between e and u are computed and displayed for up to lag 25. Also displayed are 99% confidence intervals for these variables, assuming that e is indeed white and independent of u.

The rule is that if the correlation functions go significantly outside these confidence intervals, do not accept the corresponding model as a good description of the system. Some qualifications of this statement are necessary:

- Model structures like the OE structure (Equation 3-17) and methods like the IV method (Equation 3-41) focus on the dynamics G and less about the disturbance properties H. If you are interested primarily in G, focus on the independence of e and u rather than the whiteness of e.
- Correlation between e and u for negative lags, or current $e(t)$ affecting future $u(t)$, is an indication of output feedback. This is not a reason to reject the model. Correlation at negative lags is of interest, because certain methods do not work well when feedback is present in the input-output data (see "Feedback in Data" on page 3-83), but concentrate on the positive lags in the cross-correlation plot for model validation purposes.
- When you are using the ARX model (Equation 3-14), the least squares procedure automatically makes the correlation between $e(t)$ and $u(t-k)$ zero for $k = nk, nk+1, \ldots nk+nb-1$, for the data used for the estimation.

The residuals e together with the input u are returned by

```
E = resid(Model,Data)
```

as an iddata object. As part of the validation process, you can graph the residuals using

```
plot(E)
```

for a simple visual inspection of irregularities and outliers. (See also "Outliers and Bad Data; Multiple-Experiment Data" on page 3-81.)

Model Error Models

The residual call

```
E = resid(Model,Data)
```

returns the iddata object e, which has the inputs in Data as inputs and the prediction errors (residuals) as outputs. Building models using e will thus reveal whether there is any significant influence from u to e left in the data.

Such models are called model error models, and examining them is a good complement to traditional residual analysis.

```
E= resid(Model,Data)
impulse(E,'sd',3) % An alternative to residual analysis
bode(spa(E),'sd',3) % Shows the frequency ranges
                    % with significant model errors
m = arx(E,[0 10 0])
bode(m,'sd',3)
```

Note that the `resid` command has several options to display model error properties rather than correlation functions.

Noise-Free Simulations

To check whether a model is capable of reproducing the observed output when driven by the actual input, you can run a simulation.

```
u = Data(:,[],:) % Extracting the input from the data
yh = sim(Model,u)
y = Data(:,:,[]) % Extracting the output from the data
plot(y,yh)
```

The same result is obtained by

```
compare(Data,Model)
```

It is a much tougher and more revealing test to perform this simulation, as well as the residual tests, on a fresh data set `Data` that was not used for the estimation of the model `Model`. This is called *cross validation*.

Assessing the Model Uncertainty

The estimated model is always uncertain, due to disturbances in the observed data and the lack of an absolutely correct model structure. The variability of the model that is due to the random disturbances in the output is estimated by most of the estimation procedures, and it can be displayed and illuminated in a number of ways. This variability answers the question of how different can the model be if the identification procedure is repeated, using the same model structure, but with a different data set that uses the same input sequence. It does not account for systematic errors due to an inadequate choice of model structure. There is no guarantee that the true system lies in the confidence interval. The rule is that the model should pass a residual analysis (see

"Residual Analysis" on page 3-73) test (correlation functions essentially inside the confidence lines) for the uncertainty bounds to be regarded as reliable.

The uncertainty in the different model views is displayed if the argument `'sd'` is included in the argument list,

```
command(Model,'sd',sd)
```

as explained in "Graphs of Model Properties" on page 3-64.

The uncertainty in the time response is displayed by

```
simsd(Model,u)
```

Ten possible models are drawn from the asymptotic distribution of the model `Model`. The response of each of them to the input `u` is graphed on the same diagram.

The uncertainty of these responses concerns the external input-output properties of the model. It reflects the effects of inadequate excitation and the presence of disturbances.

You can also directly get the standard deviation of the simulated result by

```
[ysim,ysimsd] = sim(Model,u)
```

The uncertainty in internal representations is manifested in the covariance matrix of the estimated parameters

```
Model.CovarianceMatrix
```

which is used to give the standard deviations of all model characteristics. The parametric uncertainty is directly available as

```
Model.da
```

for the standard deviations of `Model.a`.

Note that state-space models, estimated in a free parameterization, do not have well-defined standard deviations of the matrix elements. The model still has stored the uncertainty of the input-output behavior, so other model representations and graphs will show the uncertainty. For a state-space model in a free parameterization, it is possible to first transform it to a canonical parameterization and then display the matrix parameter uncertainties:

```
Model = pem(Data,5)
Modelc = Model
```

```
Modelc.ss = 'canon'
Modelc.da
```

All routines for computing and displaying model characteristics can also calculate and show the uncertainties. See “Transformations to Other Model Representations” on page 3-67.

Large uncertainties in these representations are caused by excessively high model orders, inadequate excitation, or bad signal-to-noise ratios.

Comparing Different Models

It is a good idea to display the model properties in terms of quantities that have more physical meaning than the parameters themselves. Bode plots, pole-zero plots, and model simulations all give a sense of the properties of the system that have been picked up by the model.

If several models of different characters give very similar Bode plots in the frequency range of interest, you can be fairly confident that these must reflect features of the true, unknown system. You can then choose the simplest model among these.

A typical identification session includes estimation in several different structures, and comparisons of the model properties. Here is an example.

```
a1 = arx(Data,[1 2 1]);
g = spa(Data);
bode(g,a1)
bode(g('n'),a1('n'))% the output disturbance spectra
am2 = armax(Data,[2 2 2 1]);
bode(g,am2)
pzmap(a1,am2,'sd',3)
```

Selecting Model Structures for Multivariable Systems

A multivariable (MIMO) system is a system with several input and output channels. All model structures in the toolbox support models with one output and several inputs. Polynomial models, `idpoly`, do not handle multioutput models, however.

Model Structures

Multivariable systems offer a potentially richer internal structure. The easiest approach, in the black-box situation, is to think just in terms of input delays and state-space model order.

A recommended approach is to get an idea of input delays from the nonparametric impulse response estimate and determine the vector `nk = [nk1,nk2,...,nkm]` where `nkj` is the minimal delay from input `j` to any of the output channels. Then try state-space models with several orders and with these delays.

```
impulse(Data,'sd',3)
Model = n4sid(Data(1:500),'nx',1:10,'nk',nk)
compare(Data(501:1000),Model)
```

The `compare` plot will reveal which output channels are easy and which are difficult to reproduce.

An alternative to find the delays is to first estimate a parametric model with delays `1`, and then examine the impulse responses of this model and determine the delays.

```
Model = pem(Data) % This uses 'best' model order.
impulse(Model,'sd',3)
Model = pem(Data,'nx',1:10,'nk',nk)
```

To test models with delay `0` in a similar way, use

```
Model = pem(Data,'best','nk',zeros(size(nk)))
```

Significant responses at delay `0` must be examined with care, because they might be caused by feedback.

Note that delays `nk` larger than `1` are incorporated in the model structure, and thus increase the state-space model order from the nominal one with `sum(max(nk-1,zeros(size(nk))))`. An alternative is to use the property `'InputDelay'`. This leads to a model that has the same delays as for `'nk'`. These are not explicitly shown in the model matrices, but stored as a property to be used when necessary. See “nk and InputDelay” on page 3-104. See also the properties of `idss` on the reference page.

If you have detailed knowledge about which orders and delays are reasonable in the different input/output channels, you can use multivariable ARX models

in the `idarx` model format. This allows you to define the orders of the input and output lags, as well as the delays, independently for the different channels.

Black-box parameterizations of multivariable systems require many parameters. Therefore, it might be important to incorporate any essential structure knowledge based on physical insight. You typically do this with continuous-time, custom model parameterizations using structured `idss` or `idgrey` models. See "Structured State-Space Models with Free Parameters: the idss Model" on page 3-48 and "State-Space Models with Coupled Parameters: the idgrey Model" on page 3-51.

Channel Selection

A particular aspect of multivariable models is the selection of channels. Models for subselections of input-output channels can be quite useful and informative. Generally speaking the models become better when more input channels are used, and worse when more output channels are used. The latter observation is due to the fact that such models have more to explain.

If you build models with several outputs and find, using `compare`, a certain output channel to be difficult to reproduce, then try to build a model of this channel alone. This will reveal if there are inherent difficulties with this output, or if it is just too difficult to handle it together with other outputs.

Analogously, if you see that using, for example, `step` or `impulse`, a certain input channel seems to have an insignificant influence on the outputs, then remove that channel, and examine whether the corresponding model becomes any worse, for example, in the `compare` plots.

The toolbox's data and model objects give full support for the bookkeeping required for these channel subselections. You select channels by direct subreferencing, and the `InputName` and `OutputName` properties form the basis for a correct combination of channels. The subreferencing follows:

```
Data(Samples,Outputs,Inputs)
Model(Outputs,Inputs)
```

Typical command sequences can be

```
Date = Data(1:500)
Datv = Data(501:1000)
m = pem(Date)
compare(Datv,m)
m1 = pem(Date(:,3,4))
compare(Datv,m,m1)
bode(m,m1)
compare(Datv,m(:,4),m1)
```

Dealing with Data

Extracting information from data is not an entirely straightforward task. In addition to the decisions required for model structure selection and validation, the data might need to be handled carefully. This section gives some advice on handling several common situations.

Offset Levels

When the data has been collected from a physical plant, it is typically measured in physical units. The levels in these raw input and output measurements might not match in any consistent way. This will force the models to waste some parameters correcting the levels.

Typically, linearized models are sought around some physical equilibrium. In such cases offsets are easily dealt with: subtract the mean levels from the input and output sequences before the estimation. It is best if the mean levels correspond to the physical equilibrium, but if such values are not known, use the sample means.

```
Data = detrend(Data);
```

Section 14.1 in Ljung (1999) discusses this in more detail. There are situations when it is not advisable to remove the sample means. It could be, for example, that the physical levels are built into the underlying model, or that integrations in the system must be handled with the right level of the input being integrated.

With the `detrend` command, you can also remove piecewise linear trends.

Outliers and Bad Data; Multiple-Experiment Data

Real data are also subject to possible bad disturbances: an unusually large disturbance, a temporary sensor or transmitter failure, etc. It is important that such outliers are not allowed to affect the models too strongly.

The robustification of the error criterion (described under `LimitError` in `Algorithm Properties` on page 4-15) helps here, but it is always good practice to examine the residuals for unusually large values, and to go back and critically evaluate the original data responsible for the large values. If the raw data is obviously in error, it can be smoothed and the estimation procedure repeated.

Often the data has portions with bad behavior. This can, for example, be due to big disturbances or sensor failures over a period of time. It can also be that there are time periods where nothing happens, the input is not exciting, etc. Then the best alternative is to break up the data into pieces of informative portions. By merging the pieces into a multiple-experiment `iddata` object, they can still be used together to build models. Another situation when multiple-experiment data is useful is when several different experiments have been performed on the same plant. The estimation routines take proper action to handle the different pieces. All estimation, simulation, and validation routines in the toolbox handle multiple-experiment data in a transparent fashion. A typical string of commands could be

```
plot(Data)
Datam = merge(Data(1:340),Data(500:897), ...
              Data(1001:1200),Data(1550:2000))
m =pem(getexp(Datam,[1,2,4])) % Portions 1, 2, and 4 for
estimation
compare(getexp(Datam,3),m) % Portion 3 for validation
```

Missing Data

In practice it is often the case that certain measurement samples are missing. The reason might be sensor failures or data acquisition failures. It might be that the data are directly reported as missing, or that plots reveal that some values are obviously in error. This can apply both to inputs and outputs. In these cases, replace the missing data by `NaN`s when forming the signal matrices and the `iddata` object. The routine `misdata` can then be applied to reconstruct the missing data in a reasonable way.

```
dat = iddata(y,u,0.2) % y and/or u contain NaNs for missing data.
dat1 = misdata(dat);
plot(dat,dat1) % Checking how the missing data
               % has been estimated in dat1
m = pem(dat1) % Model estimated using reconstructed missing data
```

See Section 14.2 in Ljung (1999) for a discussion on missing data.

Filtering Data: Focus

Depending upon the application, interest in the model can be focused on specific frequency bands. Filtering the data before the estimation, through filters that enhance these bands, improves the fit in the interesting regions.

This is accomplished in the System Identification Toolbox by the property `'Focus'`. For example, to enhance the fit in the frequency band between 0.05 and 1 rad/s, execute one of the following:

```
m = pem(Data,3,'Foc',[0.05 1])

ma = arx(Data,[2 3 1],'Foc',[0.05 1])
```

For time-domain data, this computes and uses a fifth-order Butterworth bandpass filter with passband between the indicated frequencies. For frequency-domain data, this selects the frequencies in the passband. The data is filtered through the filter before fitting the transfer function from the measured inputs (G in (Equation 3-58)) to the outputs. The disturbance model (H) is, however, estimated using the unfiltered data. Chapter 14 in Ljung (1999) discusses the role of filtering in more detail.

For several passbands, use a matrix with two columns as focus, where each row defines a passband.

For a model that does not use a disturbance description (that is, $H = 1$ in (Equation 3-58), which corresponds to `K = 0` for state-space, and `na = nc = nd = 0` for polynomial models), the `Focus` effect is the same as applying the routine to filtered data. That is,

```
m = pem(Data,3,'Foc',[0.05 1],'dist','none')
Df = idfilt(Data,[0.05 1]);
m = pem(Df,3,'dist','none')
```

give the same model.

The System Identification Toolbox contains other useful commands for related problems. For example, if you want to lower the sampling rate by a factor of 5, use

```
Dat5 = resample(Data,1,5);
```

Feedback in Data

If the system was operating in closed loop (feedback from the past outputs to the current input) when the data was collected, you must exercise some care.

Basically, all the prediction error methods work equally well for closed-loop data. Note, however, that the output-error model (Equation 3-17) and the Box-Jenkins model (Equation 3-18) are normally capable of giving a correct description of the dynamics G, even if H (which equals `1` for the output-error

model) does not allow a correct description of the disturbance properties. This is not true for closed-loop data, so you need to model the disturbance properties more carefully. Another thing to be cautious about is that impulse response effects at delay `0` very well could be traced to the feedback mechanism and not to the system itself.

The spectral analysis method and the instrumental variable techniques (with default instruments) as well as `n4sid` can give unreliable results when applied to closed-loop data. Avoid these techniques when feedback is present.

To detect whether feedback is present, use the basic method of applying `impulse` to estimate the impulse response. Significant values of the impulse response at negative lags are a clear indication of feedback. There is also a command, `feedback`, that can be applied to the data for direct tests.

When a parametric model has been estimated and the `resid` command is applied, a graph of the correlation between residuals and inputs is given. Significant correlation at negative lags again indicates output feedback in the generation of the input. Testing for feedback is, therefore, a natural part of model validation.

The `advice` function applied both to data and to estimated models will also indicate possible feedback effects in the data. See the reference page for `feedback`.

Delays

The selection of the delay `nk` in the model structure is a very important step in obtaining good identification results. You can get an idea about the delays in the system by the impulse response estimate from `impulse`.

Incorrect delays are also visible in parametric models. Underestimated delays (`nk` too small) show up as small values of leading b_k estimates compared to their standard deviations. Overestimated delays (`nk` too large) are usually visible as a significant correlation between the residuals and the input at the lags corresponding to the missing b_k terms in the `resid` plot.

A good procedure is to start by using `arxstruc` to test all feasible delays together with a second-order model. Use the delay that gives the best fit for further modeling. When you have found an otherwise satisfactory structure, vary `nk` around the nominal value within the structure and evaluate the results.

The command `delayest` directly estimates the delay, based on the `arxstruc` command.

Recursive Parameter Estimation

In many cases it might be necessary to estimate a model online at the same time as the input-output data is received. You might need the model to make some decision online, as in adaptive control, adaptive filtering, or adaptive prediction. It might be necessary to investigate possible time variation in the system's (or signal's) properties during the collection of data. Terms like *recursive identification*, *adaptive parameter estimation*, *sequential estimation*, and *online algorithms* are used for such algorithms. Chapter 11 in Ljung (1999) deals with such algorithms in some detail.

Basic Algorithm

A typical recursive identification algorithm is

$$\hat{\theta}(t) = \hat{\theta}(t-1) + K(t)(y(t) - \hat{y}(t)) \quad \textbf{(3-60)}$$

Here $\hat{\theta}(t)$ is the parameter estimate at time t, and $y(t)$ is the observed output at time t. Moreover, $\hat{y}(t)$ is a prediction of the value $y(t)$ based on observations up to time $t-1$ and also based on the current model (and possibly also earlier ones) at time $t-1$. The gain $K(t)$ determines in what way the current prediction error $y(t) - \hat{y}(t)$ affects the update of the parameter estimate. It is typically chosen as

$$K(t) = Q(t)\psi(t) \quad \textbf{(3-61)}$$

where $\psi(t)$ is (an approximation of) the gradient with respect to θ of $\hat{y}(t|\theta)$. The latter symbol is the prediction of $y(t)$ according to the model described by θ. Note that model structures like AR and ARX that correspond to linear regressions can be written as

$$y(t) = \psi^T(t)\theta_0(t) + e(t) \quad \textbf{(3-62)}$$

where the *regression vector* $\psi(t)$ contains old values of observed inputs and outputs, and $\theta_0(t)$ represents the true description of the system. Moreover, $e(t)$ is the noise source (the innovations). Compare with (Equation 3-14). The natural prediction is $\hat{y}(t) = \psi^T(t)\hat{\theta}(t-1)$, and its gradient with respect to θ becomes exactly $\psi(t)$.

For models that cannot be written as linear regressions, you cannot recursively compute the exact prediction and its gradient for the current estimate $\theta(t-1)$. Then you must use approximations $\hat{y}(t)$ and $\psi(t)$ instead. Section 11.4 in Ljung (1999) describes suitable ways of computing such approximations for general model structures.

The matrix $Q(t)$, which affects both the adaptation gain and the direction in which the updates are made, can be chosen in several different ways. This is discussed in the following.

Choosing an Adaptation Mechanism and Gain

The most logical approach to the adaptation problem is to assume a certain model for how the true parameters θ_0 change. A typical choice is to describe these parameters as a random walk.

$$\theta_0(t) = \theta_0(t-1) + w(t) \tag{3-63}$$

Here $w(t)$ is assumed to be white Gaussian noise with covariance matrix

$$Ew(t)w^T(t) = R_1 \tag{3-64}$$

Suppose that the underlying description of the observations is a linear regression (Equation 3-62). An optimal choice of $Q(t)$ in (Equation 3-60) and (Equation 3-61) can then be computed from the Kalman filter, and the complete algorithm becomes

$$\begin{aligned}
\hat{\theta}(t) &= \hat{\theta}(t-1) + K(t)(y(t) - \hat{y}(t)) \\
\hat{y}(t) &= \psi^T(t)\hat{\theta}(t-1) \\
K(t) &= Q(t)\psi(t) \\
Q(t) &= \frac{P(t-1)}{R_2 + \psi(t)^T P(t-1)\psi(t)} \\
P(t) &= P(t-1) + R_1 - \frac{P(t-1)\psi(t)\psi(t)^T P(t-1)}{R_2 + \psi(t)^T P(t-1)\psi(t)}
\end{aligned} \tag{3-65}$$

Here R_2 is the variance of the innovations $e(t)$ in (Equation 3-62):

$R_2 = Ee^2(t)$ (a scalar). The algorithm (Equation 3-65) is called the *Kalman*

filter (KF) approach to adaptation, with *drift matrix* R_1. See Equations (11.66) and (11.67) in Ljung (1999). The algorithm is entirely specified by $R_1, R_2, P(0), \theta(0)$, and the sequence of data $y(t), \psi(t)$, $t = 1, 2$. Even though the algorithm is appropriate for a linear regression model structure, it can also be applied in the general case where $\hat{y}(t)$ is computed in a different way from (Equation 3-65b).

Another approach is to discount old measurements exponentially, so that an observation that is τ samples old carries a weight that is λ^{τ} of the weight of the most recent observation. This means that the following function is minimized rather than (Equation 3-39) at time *t*:

$$\sum_{k=1}^{t} \lambda^{t-k} e^2(k) \qquad \textbf{(3-66)}$$

Here λ is a positive number (slightly) less than 1. The measurements that are older than $\tau = 1/(1-\lambda)$ carry a weight in the expression above that is less than about 0.3. Think of $\tau = 1/(1-\lambda)$ as the *memory horizon* of the approach. Typical values of λ are in the range 0.97 to 0.995.

The criterion (Equation 3-66) can be minimized exactly in the linear regression case giving the algorithm (Equation 3-65abc) with the following choice of $Q(t)$:

$$\begin{aligned} Q(t) &= P(t) = \frac{P(t-1)}{\lambda + \psi(t)^T P(t-1)\psi(t)} \\ P(t) &= \frac{1}{\lambda}\left(P(t-1) - \frac{P(t-1)\psi(t)\psi(t)^T P(t-1)}{\lambda + \psi(t)^T P(t-1)\psi(t)} \right) \end{aligned} \qquad \textbf{(3-67)}$$

This algorithm is called the *forgetting factor* (FF) approach to adaptation, with the *forgetting factor* λ. See Equation (11.63) in Ljung (1999). The algorithm is also known as *recursive least squares* (RLS) in the linear regression case. Note that $\lambda = 1$ in this approach gives the same algorithm as $R_1 = 0, R_2 = 1$ in the Kalman filter approach.

A third approach is to allow the matrix $Q(t)$ to be a multiple of the identity matrix.

$$Q(t) = \gamma I \qquad \textbf{(3-68)}$$

It can also be normalized with respect to the size of ψ.

$$Q(t) = \frac{\gamma}{|\psi(t)|^2} I \qquad \textbf{(3-69)}$$

See Equations (11.45) and (11.46), respectively, in Ljung (1999). These choices of $Q(t)$ move the updates of $\hat{\theta}$ in (Equation 3-60) in the negative gradient direction (with respect to θ) of the criterion (Equation 3-39). Therefore, (Equation 3-68) is called the *unnormalized gradient* (UG) approach and (Equation 3-69) the *normalized gradient* (NG) approach to adaptation, with gain γ. The gradient methods are also known as *least mean squares* (LMS) in the linear regression case.

Available Algorithms

The System Identification Toolbox provides the following functions that implement all common recursive identification algorithms for model structures in the family (Equation 3-43): `rarmax`, `rarx`, `rbj`, `rpem`, `rplr`, and `roe`. They all share the following basic syntax:

```
[thm,yh] = rfcn(z,nn,adm,adg)
```

Here `z` contains the output-input data as usual. `nn` specifies the model structure, exactly as for the corresponding offline algorithm. The arguments `adm` and `adg` select the adaptation mechanism and adaptation gain listed above.

```
adm = 'ff'; adg = lam
```

gives the forgetting factor algorithm (Equation 3-67), with forgetting factor `lam`.

```
adm = 'ug'; adg = gam
```

gives the unnormalized gradient approach (Equation 3-68) with gain `gam`. Similarly,

```
adm = 'ng'; adg = gam
```

gives the normalized gradient approach (Equation 3-69). To obtain the Kalman filter approach (Equation 3-65) with drift matrix `R1`, enter

```
adm = 'kf'; adg = R1
```

The value of R_2 is always 1. Note that the estimates $\hat{\theta}$ in (Equation 3-65) are not affected if all the matrices R_1, R_2 and $P(0)$ are scaled by the same number. Therefore you can always scale the original problem so that R_2 becomes 1.

The output argument `thm` is a matrix that contains the current models at the different samples. Row `k` of `thm` contains the model parameters, in alphabetical order at sample time k, corresponding to row `k` in the data matrix `z`. The ordering of the parameters is the same as `m.par` would give when applied to a corresponding offline model.

The output argument `yh` is a column vector that contains, in row `k`, the predicted value of $y(k)$, based on past observations and current model. The vector `yh` thus contains the adaptive predictions of the outputs, and is useful also for noise canceling and other adaptive filtering applications.

The functions also have optional input arguments that allow the specification of $\theta(0), P(0)$, and $\psi(0)$. Optional output arguments include the last value of the matrix P and of the vector ψ.

Now, `rarx` is a recursive variant of `arx`; similarly `rarmax` is the recursive counterpart of `armax`, and so on. Note, however, that `rarx` does not handle multioutput systems, and `rpem` does not handle state-space structures.

The function `rplr` is a variant of `rpem`, and uses a different approximation of the gradient ψ. It is known as the *recursive pseudolinear regression approach*, and contains some well-known special cases. See Equation (11.57) in Ljung (1999). When applied to the output-error model (`nn=[0 nb 0 0 nf nk]`) it results in methods known as HARF (`'ff'-case`) and SHARF (`'ng'-case`). The common *extended least squares* (ELS) method is an `rplr` algorithm for the ARMAX model (`nn=[na nb nc 0 0 nk]`).

The following example shows a second-order output-error model, which is built recursively, and its time-varying parameter estimates plotted as functions of time.

```
thm = roe(z,[2 2 1],'ff',0.98);
plot(thm)
```

The next example shows how a second-order ARMAX model is recursively estimated by the ELS method, using Kalman filter adaptation. The resulting static gains of the estimated models are then plotted as a function of time.

```
[N,dum]=size(z);
thm = rplr(z,[2 2 2 0 0 1],'kf',0.01*eye(6));
```

```
nums = sum(thm(:,3:4)')';
dens = ones(N,1)+sum(thm(:,1:2)')';
stg = nums./dens;
plot(stg)
```

So far, the examples of applications where a batch of data is examined cover studies of the variability of the system. The algorithms are, however, also appropriate for true online applications, where the computed model is used for some online decision. You do this by storing the update information in $\hat{\theta}(t-1), P(t-1)$ and information about past data in $\phi(t-1)$ (and $\psi(t-1)$) and using that information as initial data for the next time step. The following example shows the recursive least squares algorithm being used online (just to plot one current parameter estimate).

```
% Initialization, first i/o pair y,u (scalars)
[th,yh,P,phi] = rarx([y u],[2 2 1],'ff',0.98);
axis([1 50 -2 2])
plot(1,th(1),'*'),hold
%The online loop:
for k = 2:50
% At time k receive y,u
[th,yh,P,phi] = rarx([y u],[2 2 1],'ff',0.98,th',P,phi);
plot(k,th(1),'*')
end
```

Execute `iddemo #10` to illustrate the recursive algorithms.

Segmentation of Data

Sometimes the system or signal exhibits abrupt changes during the time when the data is collected. It might be important in certain applications to find the time instants when the changes occur and to develop models for the different segments during which the system does not change. This is the *segmentation problem*. Fault detection in systems and detection of trend breaks in time series can serve as two examples of typical problems.

The System Identification Toolbox offers the function `segment` to deal with the segmentation problem. The basic syntax is

```
thm = segment(z,nn)
```

with a format like `rarx` or `rarmax`. The matrix `thm` contains the piecewise constant models in the same format as for the algorithms described earlier in this section.

The algorithm that is implemented in `segment` is based on a model description like (Equation 3-63), where the change term $w(t)$ is zero most of the time, but now and then it abruptly changes the system parameters $\theta_0(t)$. Several Kalman filters that estimate these parameters are run in parallel, each of them corresponding to a particular assumption about when the system actually changed. The relative reliability of these assumed system behaviors is constantly judged, and unlikely hypotheses are replaced by new ones. Optional arguments allow the specification of the measurement noise variance R_2 in (Equation 3-62), the probability of a jump, the number of parallel models in use, and also the guaranteed lifespan of each hypothesis. See the `segment` reference page.

Miscellaneous Topics

This section describes a number of miscellaneous topics. Most of the information here is also covered in other parts of the manual, but since manuals seldom are read from the beginning, you can also check whether a particular topic is brought up here.

- "Time-Series Modeling" on page 3-93
- "Periodic Inputs" on page 3-96
- "Connections Between the Control System Toolbox and the System Identification Toolbox" on page 3-96
- "Memory/Speed Tradeoffs" on page 3-98
- "Local Minima" on page 3-98
- "Initial Parameter Values" on page 3-99
- "Initial State" on page 3-100
- "Initial States for Frequency Domain Data" on page 3-101
- "Using Simulation to Validate Estimated Models" on page 3-101
- "The Estimated Parameter Covariance Matrix" on page 3-103
- "No Covariance" on page 3-104
- "nk and InputDelay" on page 3-104
- "Linear Regression Models" on page 3-106
- "Spectrum Normalization and the Sampling Interval" on page 3-107
- "Interpretation of the Loss Function" on page 3-109
- "Enumeration of Estimated Parameters" on page 3-110
- "Complex-Valued Data" on page 3-111
- "Strange Results" on page 3-111

Time-Series Modeling

When there is no input present, the general model (Equation 3-43) reduces to the ARMA model structure.

$$A(q)y(t) = C(q)e(t)$$

With $C(q) = 1$ you have an AR model structure.

Similarly, a state-space model for a time series is given by

$$x(t+1) = Ax(t) + Ke(t)$$
$$y(t) = Cx(t) + e(t)$$

so that the matrices B and D are empty.

Basically all commands still apply to these time-series models, but with natural modifications. They are listed as follows:

```
m= idpoly(A,[ ],C)
e = iddata([],idinput(300,'rgs'))
y = sim(m,e)
```

If a time series s is given as a vector or a matrix, it is put into the `iddata` format by

```
y = iddata(s,[],Ts);
```

Spectral analysis (`etfe` and `spa`) returns results in the `idfrd` model format, which now just contains `SpectrumData` and its variance. `bode` will only plot these signal spectra and, if required, the confidence intervals.

```
g = spa(y)
p= etfe(y)
bode(g,p,'sd',3)
```

Note that `etfe` gives the *periodogram* estimate `p` of the spectrum.

`armax` and `arx` work the same way, but need no specification of `nb` and `nk`.

```
th = arx(y,na)
th = armax(y,[na nc])
```

Note that `arx` also handles multivariable signals, and so do `n4sid` and `pem`.

```
m = n4sid(y) % default order
bode(m)
compare(y,m,10) % 10-step ahead predictions being evaluated.
```

You can build structured state-space models of time series simply by specifying `B = []`, `D = []` in `idss` and `idgrey`. `resid` works the same way for time-series models, but does not provide any input-residual correlation plots.

```
resid(m,y)
```

In addition there are two commands that are specifically constructed for building scalar AR models of time series. One is

```
m = ar(y,na)
```

which has an option that allows you to choose the algorithm from a group of several popular techniques for computing the least squares AR model. Among these are Burg's method, a geometric lattice method, the Yule-Walker approach, and a modified covariance method. See Chapter 4, "Functions — By Category" for details. The other command is

```
m = ivar(y,na)
```

which uses an instrumental variables technique to compute the AR part of a time series.

Finally, when no input is present, the functions `bj`, `iv`, `iv4`, and `oe` are not of interest.

Here is an example where you can simulate a time series, compare spectral estimates and covariance function estimates, and also the predictions of the model.

```
ts0 = idpoly([1 -1.5 0.7],[]);
ir = sim(ts0,[1;zeros(24,1)]);
Ry0 = conv(ir,ir(25:-1:1)); % The true covariance function
e = idinput(200,'rgs');
y = sim(ts0,e); % y is a vector here
y = iddata(y) % iddata object with sampling time 1.
plot(y)
per = etfe(y);
speh = spa(y);
ffplot(per,speh,ts0)
ts2 = ar(y,2); % A second-order AR model:
ffplot(speh,ts2,ts0,'sd',3)
% The covariance function estimates:
Ryh = covf(y,25);
Ryh = [Ryh(end:-1:2),Ryh]';
ir2 = sim(ts2,[1;zeros(24,1)]);
Ry2 = conv(ir2,ir2(25:-1:1));
plot([-24:24]'*ones(1,3),[Ryh,Ry2,Ry0])
% The prediction ability of the model:
compare(y,ts2,5)
```

Periodic Inputs

It is often an advantage to use a periodic input for identification whenever possible. See Section 13.3 in Ljung (1999). If you import or create a periodic input, as in

```
u = idinput([300 2 5]) % Period 300, 2 inputs, 5 periods
```

you should set the corresponding period in the `iddata` object.

```
u = iddata([],u,'Period',[300; 300]);
```

Normally, an even number of periods should be represented in the data. That allows the estimation routines to do the right things. For example, when called with data with periodic inputs, `etfe` honors the period and computes the frequency response on a suitably chosen frequency grid. Try this:

```
m0 =idpoly([1 -1.5 0.7],[0 1 0.5]);
u = idinput([10 1 150],'rbs');
u = iddata([],u,'Period',10);
e = iddata([],randn(1500,1));
y = sim(m0, [u e])
g = etfe([y u])
bode(g,'x',m0) % Good fit at the 5 excited frequencies
```

Connections Between the Control System Toolbox and the System Identification Toolbox

The objects and functions of the Control System Toolbox are quite similar to those of the System Identification Toolbox. This means that the two toolboxes can be run together efficiently.

Function Calls

The function calls are the same for many essential functions. `bode`, `freqresp`, `impulse`, `minreal`, `nyquist`, `ssdata`, `step`, `tfdata`, `zpkdata`, etc., all do the same things with essentially the same syntax. The System Identification Toolbox commands, however, also handle model uncertainty. The System Identification Toolbox commands are used whenever at least one of the objects in the argument list is an `idmodel` or `idfrd` object.

Subreferencing of channels and concatenations also follow the same syntax.

Moreover, most of the LTI commands for model manipulation, like `append`, `augstate`, `balreal`, `canon`, `feedback`, `G1+G2`, `G1*G2`, etc., will work (using the Control System Toolbox) in the expected way, returning `idmodel` objects. However, in most cases covariance information is lost.

Object Relations

Because the System Identification Toolbox can be run without the Control System Toolbox, there are no formal parent/child relations between the objects in the two toolboxes. There are, however, easy transformations between them. The command that creates `idmodel`, `idss`, and `idpoly` will accept any LTI object, `zpk`, `tf`, or `ss`. `idfrd` can similarly be created from `frd` objects. If the LTI object has an `InputGroup` named `'noise'` these inputs will be treated as normalized white noise when you are creating the `idmodel` object with correct disturbance model information.

Analogously, `ss`, `zpk`, `tf`, and `frd` accept any `idmodel` or `idfrd` (in the case of `frd`) object. The covariance information is then not stored in the LTI objects, but all disturbance information is translated to a group of extra input channels with the group name `'noise'`. If these are interpreted as normalized white noise, the LTI objects have the same disturbance properties as the original `imdmodel` object.

These simple relations also mean that it is easy to use any LTI command in the Control System Toolbox and return to System Identification Toolbox objects.

```
Mb = idss(balreal(ss(M)))
```

Plot Relations

Although the calls `bode`, `step`, etc., have essentially the same syntax, the plots look different. The System Identification Toolbox commands show confidence regions when required, and typically show the different input/output channels as separate plots. The sorting of the channels is based on the `InputName` and `OutputName` properties. Therefore the System Identification Toolbox commands allow any mix of models, not necessarily of the same sizes.

The System Identification Toolbox plot commands do not offer the same options and plot interaction facilities as `ltiview`. However, applying `view` to one or several `idmodel` objects invokes the LTI Viewer.

Here is an example of the interplay between the functions in the two toolboxes.

```
m0 = drss(4,3,2)
m0 = idss(m0,'NoiseVar',0.1*eye(3))
u = iddata([], idinput([800 2],'rbs'));
e = iddata([], randn(800, 3));
y = sim(m0, [u e])
Data = [y u];
m = pem(Data(1:400))
tf(m)
compare(Data(401:800),m)
view(m)
```

Memory/Speed Tradeoffs

On machines with no formal memory limitations, it is still of interest to monitor the sizes of the matrices that are formed. The typical situation is when an overdetermined set of linear equations is solved for the least squares solution. The solution time depends, of course, on the dimensions of the corresponding matrix. The number of rows corresponds to the number of observed data, while the number of columns corresponds to the number of estimated parameters. The property `MaxSize` used with all the relevant M-files, prevents, whenever possible, the formation of matrices with more than `MaxSize` elements. Larger data sets and/or higher-order models are handled by `for` loops. `for` loops give a linear increase in time when the data record is increased, plus some overhead.

If you regularly work with large data sets and/or high-order models, it is advisable to tailor the memory and speed tradeoff to your machine by choosing `MaxSize` carefully. You could also change the default value of `MaxSize` in the M-file `idmsize`. Then the default value of `MaxSize` (that is, `'Auto'`) will be tailored to your needs. Note that this value is allowed to depend on the number of rows and columns of the matrices formed.

Local Minima

The iterative search procedures in `pem`, `armax`, `oe`, and `bj` lead to models corresponding to a local minimum of the criterion function (Equation 3-39). Nothing guarantees that this local minimum is also a global minimum. The startup procedure for black-box models in these routines is, however, reasonably efficient in giving initial estimates that lead to the global minimum.

If there is an indication that a minimum is not as good as you expected, try starting the minimization at several different initial conditions, to see if a smaller value of the loss function can be found. You can use the function `init` for that.

Initial Parameter Values

When only orders and delays are specified, the functions `armax`, `bj`, `oe`, and `pem` use a startup procedure to produce initial values. The startup procedure goes through two to four least squares and instrumental variable steps. It is reasonably efficient in that it usually saves several iterations in the minimization phase. Sometimes, however, it might pay to use other initial conditions. For example, you can use an `iv4` estimate computed earlier as an initial condition for estimating an output-error model of the same structure.

```
m1 = iv4(Data,[na nb nk]);
set(m1,'a',1,'f',m1.a)
m2= oe(Data,m1);
```

Another example is when you want to try a model with one more delay (for example, three instead of two) because the leading b-coefficient is quite small.

```
m1 = armax(Data,[3 3 2 2]);
m1.b(3) = 0
m2 = armax(Data,m1);
```

If you decrease the number of delays, remember that leading zeros in the B-polynomial are treated as delays. Suppose you go from three to two delays in the above example:

```
m1 = armax(z,[3 3 2 3]);
m1.b(3) = 0.00001;
m2 = armax(Data,m1);
```

Note that when you construct homemade initial conditions, the conditions must correspond to a stable predictor (C and F being Hurwitz polynomials), and they should not contain any exact pole-zero cancellations.

For user-defined structured state-space and multioutput models, you must provide the initial parameter values (initial model) when defining the structure in `idss` or `idgrey`. The basic approach is to use physical insight to choose initial values of the parameters with physical significance, and try some

different (randomized) initial values for the others. You can use the routine `init` for that.

For free state-space parameterizations, it can sometimes be difficult to reach the global minimum. If you see that the minimization routine seems to get stuck (turn `trace` on and check the improvements per iteration), it might be a good idea to transform state-space matrices to other realizations, as in

```
m = pem(Data,5,'trace','on')
m.ss = 'can';
m = pem(Data,m);
m = balreal(m); % If you have the Control System Toolbox
m = pem(Data,m);
```

Initial State

The filter that computes the prediction errors in (Equation 3-36) needs to be properly initialized. For input-output (polynomial) models, values of inputs, outputs, and predictions prior to time $t = 0$ are required, and state-space models need the initial state $x(0)$. There are several ways to handle these unknown states. A simple one is to take all unknown values as zero. If the model predictor has slow dynamics (that is, the poles of CF or the eigenvalues of $A\text{-}KC$ are close to the unit circle), this could have a very bad effect on the parameter estimates. It is particularly pronounced for output-error models, where the noise model cannot be adjusted to handle slow transients from initial conditions.

The toolbox offers a number of options to deal with the initial state of the predictor. They are handled by the model property `InitialState`. The unknown state can be treated as a vector of unknown parameters (`InitialState = 'Estimate'`). They can be set to zero (`InitialState = 'Zero'`) or estimated by a backward prediction method (`InitialState = 'Backcast'`). They can also be fixed to any user-defined value. The default value is `InitialState = 'Auto'`, which makes an automatic choice among the options, guided by the estimation data. For details, see the `idss` and `idpoly` reference pages. Basically, the effect of the initial conditions on the prediction errors is tested, and if it seems negligible, `'zero'` is chosen, which gives a fast and efficient algorithm. Otherwise the initial state is estimated or backcast. `EstimationInfo` will contain information about which method was chosen in this case.

Proper handling of the initial state is necessary both when models are estimated and when predictions and simulations are compared. The commands `predict`, `pe`, `sim`, and `compare` all offer options for how to deal with this.

Note that the estimated initial condition `x(0)` depends on both the model and the estimation data. It is thus a characteristic that does not necessarily have relevance when the model is applied to another data set.

Initial States for Frequency Domain Data

The calculations using frequency-domain data essentially assume that the underlying time-domain data is periodic. Otherwise treating convolutions as multiplications in the frequency domain creates end-effect errors. Therefore *initial conditions* are as important for frequency-domain data as for time-domain data. The proper initial conditions in the frequency domain are those that make up for deviations in periodicity of the original data.

From a formal point of view, these initial conditions can be handled quite analogously to the time-domain case. They can be taken as zero, which is the correct choice if indeed the original data was periodic. They can also be estimated and backcast. Therefore the values of the property `InitialState` can assume the same values, `'zero'`, `'estimate'`, `'backcast'`, and `'auto'`, as in the time-domain case. This also applies to the `arx` command, for which `InitialState` has no effect for time-domain data.

Note, again, that the estimated value, `x0`, is tied to the data set for which it was estimated. In particular, you should not make any time-domain interpretation of it in case it was estimated using frequency-domain data.

Using Simulation to Validate Estimated Models

This section describes how to simulate a model in a simulation environment, such as Simulink, to verify that the simulation results match the experimental output data from a validation data set.

To simulate an estimated state-space model, you must specify the initial-state values for the validation data in the simulation. The initial states you specify for the simulation must correspond to the data set you use in the simulation.

Note The validation data need not differ from the estimation data. If you choose to use different data for validation in Simulink, you must simulate with initial states that correspond to this data set.

The `X0` model property stores the estimated initial states of the model. This value corresponds to the data that was used for estimation. If you use a different data set for validation in Simulink, you cannot use `X0` to represent the model's initial states during validation.

Tip Alternatively, you can use `compare` to perform model validation. This function automatically computes the required initial conditions by default.

When you estimate a model using a data set that consists of multiple experiments, the initial-states property `X0` stores only the estimated states corresponding to the last experiment. To validate a model using initial states from an experiment other than the last, use the `pe` function to estimate `X0` again for that specific experiment (see the following example).

Example — Validating an Estimated Model in Simulink

Suppose you estimate the three-state model `M` using a merged data set `Z`, which contains data from 5 experiments — `z1`, `z2`, `z3`, `z4`, and `z5`:

```
Z = merge(z1,z2,z3,z4,z5);
M = n4sid(Z,3);
```

When a model uses several data sets, the initial-states property stores only the estimated states corresponding to the last data set. In this example, `M.X0` is a vector of length `3` (corresponding to the three states of the model). The values of `M.X0` are the estimated state values corresponding to `z5`.

The following procedure describes how to access the initial states of `z2` for the simulation, where `z2` is a portion of the estimation data `Z`.

To specify the settings of the `idmodel` block in Simulink for comparing the measured output from experiment `z2` with the simulated output:

1. Estimate the initial states using the second experiment as input, that is `Z(z2.u)`, as follows:

   ```
   [E,XOest] = pe(M,getexp(Z,2)
   ```

   Here, the function `getexp(Z,2)` gets the data in `z2`.

2. In Simulink, open the Function Block Parameters dialog box for the `idmodel` block.

3. In the **idmodel variable** field, type `M` to specify the estimated model.

4. In the **Initial states...** field, type `XOest` to specify the estimated initial states.

5. Click **OK**.

Run the simulation with these settings to compare the measured output `z2.y` to the simulated output.

The Estimated Parameter Covariance Matrix

The estimated parameters are uncertain. The amount of uncertainty is measured and described by the covariance matrix of the estimated parameter vector (this vector is a random variable, because it depends on the random noise that has affected the output). This covariance (uncertainty) can also be estimated from data, as described, for example, in Chapter 9 of Ljung (1999). The estimated covariance matrix is contained in the estimated model as the property `Model.CovarianceMatrix`. It is used to compute all relevant uncertainty measures of various model input-output properties (Bode plots, uncertain model output, zeros and poles, etc.).

The estimate of the covariance matrix is based on the assumption that the model structure is capable of giving a correct description of the system. For models that contain a disturbance model (H is estimated), it is assumed that the model will produce white residuals, for the uncertainty estimate to be correct.

However, for output-error models (H fixed to `1`, corresponding to $K = 0$ for state-space models and $C = D = A = 1$ for polynomial models), it is not assumed

that the residuals are white. Instead, their color is estimated, and a correct estimate of the covariance estimate is used. This corresponds to Equation (9.42) in Ljung (1999).

No Covariance

Evaluating and visualizing the uncertainty of the estimated models is a very important aspect of system identification. Handling and translating covariance information takes a major part of the time in many of the routines of the System Identification Toolbox. For example, in `n4sid`, calculating the Cramer-Rao bound (which in this case is used as an indication of the covariance properties) takes much longer than estimating the actual model. In `d2c` and `c2d`, most of the time is spent on covariance handling. If you build models that are of a preliminary nature, and you would like to speed up the calculations, you can add the property name/property value pair `'Covariance'/'None'` to the list of arguments in most relevant routines. This will prevent covariance calculations and set a flag not to spend time on this in future use of the model. You can also set this flag in the model at any time by

```
Model.cov = 'no'
```

nk and InputDelay

What's the difference between the properties `nk` and `InputDelay`? `InputDelay` is defined for all `idmodel` and `idfrd` objects, while `nk` is defined for `idarx` and `idpoly` as well as for `'Free'` and `'Canonical'` `idss` models. Both properties indicate a delay from the input channels to the outputs. For `idarx`, `nk` is a matrix describing the delays in the different input/output channels, but otherwise both `nk` and `InputDelay` describe the delay from a certain input channel to all the output channels.

`InputDelay` is really a flag that tells the model to append the input delays as time lags when the model is simulated, or as phase lags when the frequency functions are computed. The `InputDelay` does not show up when the model is represented in state-space form, nor as transfer functions, nor in the input-output polynomials. `InputDelay` can be used both for continuous- and discrete-time models. In the latter case, the `InputDelay` is measured in number of samples. Moreover, `InputDelay` can assume negative values in order to handle noncausal models.

The property `nk`, on the other hand, is a model structure property, requiring the model to contain the indicated number of delays whatever the parameter

values. This means that the state-space matrices, the transfer functions, etc., will show these delays in an explicit manner. Consequently, `nk` is not defined for continuous-time models (other than as a flag for free and canonical state-space models whether a D matrix is included (`nk = 0`) or set to zero (`nk = 1`)).

Otherwise the two properties can be used in the same way. Note that the actual delay is the sum of `nk` and `InputDelay`. Therefore

```
m1 = oe(Data,[3 3 0],'InputDelay',3)
m2 = oe(Data,[3 3 1],'InputDelay',2)
m3 = oe(Data,[3 3 3]);
bode(m1,m2,m3)
```

gives identical `bode` plots (up to minor variations due to end effects in the data records). For state-space models, `nk` is 1 by default. Therefore

```
m1 = pem(Data,4,'InputDelay',[3 2 4])
m2 = pem(Data,4,'nk',[4 3 5])
bode(m1,m2)
A1 = m1.A
A2 = m2.A
```

give the same bode plots, while `A1` and `A2` are different. In fact while `A1` is of size 4-by-4, the matrix `A2` is of size 13-by-13, because nine extra states are required to accommodate the extra `3+2+4` input delays.

For continuous-time data, `nk` can only be used to flag whether a D matrix should be included in a state-space model. Any real delays must be handled by `InputDelay`. (Note that `u` is short for `input`, so you can write `udel` for `InputDelay`.)

```
Df= fft(Dt)
Df.Ts = 0: % Bandlimited data
m = oe(Df.[1 3],'udel',5); % 5 seconds delay in estimated model
```

If you build a continuous-time model from discrete-time data, you could use

```
m = pem(Dt,3,'nk',5,'sspar','can','ts',0)
```

This will build a preliminary model with a delay of five samples (using `n4sid`), which is then converted to continuous time, where the time delays are taken care of by `InputDelay`. The `pem` iterations are then carried out for this continuous-time model.

Although nk and InputDelay have the same significance for a model, there are differences in the computational aspects of the estimation process. Generally speaking, it is faster to estimate a model with a long delay using InputDelay, rather than nk, because this gives fewer states.

There is a command inpd2nk that translates a model with a nonzero InputDelay to one where the delay is handled via nk. The commands pe and predict also offer the possibility to do this transformation when estimating initial states.

Note that setting nk to a certain value for a given model gives a model structure that has the indicated delay for any parameter values. The impulse response of the model might however change (not just be shifted) by this assignment.

Linear Regression Models

A linear regression model is of the type

$$y(t) = \theta^T \varphi(t) + e(t) \qquad \textbf{(3-70)}$$

where $y(t)$ and $\varphi(t)$ are measured variables and $e(t)$ represents noise. Such models are very useful in most applications. They allow, for example, the inclusion of nonlinear effects in a simple way. The System Identification Toolbox function arx allows an arbitrary number of inputs. You can therefore handle arbitrary linear regression models with arx. For example, if you want to build a model of the type

$$y(t) = b_0 + b_1 u(t) + b_2 u^2(t) + b_3 u^3(t) \qquad \textbf{(3-71)}$$

let

```
Data = iddata(y,[ones(size(u)), u, u.^2, u.^3]);
m= arx(Data,'na',0,'nb',[1 1 1 1],'nk',[ 0 0 0 0])
```

This is formally a model with one output and four inputs, but all the model testing in terms of compare, sim, and resid operates in the natural way for the model (Equation 3-70), once the data set Data is defined as above.

Note that when pem is applied to linear regression structures, by default a robustified quadratic criterion is used. The search for a minimum of the criterion function is carried out by iterative search. Normally, you should use this robustified criterion. If you insist on a quadratic criterion, then set the argument LimitError in pem to 0. Then pem also converges in one step.

Spectrum Normalization and the Sampling Interval

In the function spa, the spectrum estimate is normalized with the sampling interval T as

$$\Phi_y(\omega) = T \sum_{k=-M}^{M} R_y(kT) e^{-i\omega T} W_M(k) \qquad \textbf{(3-72)}$$

where

$$\hat{R}_y(kT) = \frac{1}{N} \sum_{l=1}^{N} y(lT - kT) y(lT)$$

(See also (Equation 3-3).) The normalization in `etfe` is consistent with (Equation 3-72). This normalization means that the unit of $\Phi_y(\omega)$ is "power per radians/time unit" and that the frequency scale is "radians/time unit." You then have

$$Ey^2(t) = \frac{1}{2\pi} \int_{-\pi/T}^{\pi/T} \Phi_y(\omega) d\omega \qquad \textbf{(3-73)}$$

In MATLAB, therefore, you have $S1 \approx S2$, where

```
y.ts = T
sp = spa(y);
phiy = squeeze(sp.spec) % squeeze takes out the spurious
dimensions
S1 = sum(phiy)/length(phiy)/T;
S2 = sum(y.^2)/size(y,1);
```

Note that `phiy` contains $\Phi_y(\omega)$ between $\omega = 0$ and $\omega = \pi/T$ with a frequency step of ¼ `π / (T length(phiy))`. The sum `S1` is, therefore, the rectangular approximation of the integral in (Equation 3-73). The spectrum normalization differs from the one used by `spectrum` in the Signal Processing Toolbox, and the above example shows the nature of the difference.

The normalization with T (in Equation 3-72) also gives consistent results when time series are decimated. If the energy above the Nyquist frequency is

removed before decimation (as is done in `resample`), the spectral estimates coincide; otherwise you see folding effects.

Try the following sequence of commands.

```
m0 = idpoly(1,[ ],[1 1 1 1]);
       % 4th-order MA-process
e = idinput(2000,'rgs')
e = iddata([], e, 'Ts', 1);
y = sim(m0, e);
g1 = spa(y);
g2 = spa(y(1:4:2000)); % This code automatically sets Ts to 4.
ffplot(g1,g2) % Folding effects
g3 = spa(resample(y,1,4)); % Prefilter applied
ffplot(g1,g3) % No folding
```

For a parametric noise (time-series) model

$$y(t) = H(q)e(t); \qquad Ee^2(t) = \lambda$$

the spectrum is computed as

$$\Phi_y(\omega) = \lambda T\left|H(e^{i\omega T})\right|^2 \tag{3-74}$$

which is consistent with (Equation 3-72) and (Equation 3-73). Think of λT as the spectral density of the white noise source $e(t)$.

When a parametric disturbance model is transformed between continuous time and discrete time and/or resampled at another sampling rate, the functions `c2d` and `d2c` in the System Identification Toolbox use formulas that are formally correct only for piecewise constant inputs. (See (Equation 3-29).) This approximation is good when T is small compared to the bandwidth of the noise. During these transformations the variance λ of the innovations $e(t)$ is changed so that the spectral density $T \cdot \lambda$ remains constant. This has two effects:

- The spectrum scalings are consistent, so that the noise spectrum is essentially invariant (up to the Nyquist frequency) with respect to resampling.
- Simulation with noise using `sim` has a higher noise level when performed at faster sampling.

This latter effect is well in line with the standard description that the underlying continuous-time model is subject to continuous-time white noise disturbances (which have infinite, instantaneous variance), and appropriate lowpass filtering is applied before the measurements are sampled. If this effect is unwanted in a particular application, scale the noise source appropriately before applying `sim`.

Note the following cautions relating to these transformations of disturbance models. Continuous-time disturbance models must have a white noise component. Otherwise the underlying state-space model, which is formed and used in `c2d` and `d2c`, is ill-defined. Warnings about this are issued by `idpoly` and these functions. Modify the *C*-polynomial accordingly. Make the degree of the monic *C*-polynomial in continuous time equal to the sum of the degrees of the monic *A*- and *D*-polynomials, that is, in continuous time.

```
length(C)-1 = (length(A)-1)+(length(D)-1)
```

Interpretation of the Loss Function

The value of the quadratic loss function is given as the field `LossFcn` in the `EstimationInfo` of the model.

```
m.es.LossFcn
```

For multioutput systems, this is equal to the determinant of the estimated covariance matrix of the noise source *e*.

For most models, you obtain the estimated covariance matrix of the innovations by forming the corresponding sample mean of the prediction errors (squared), computed (using `pe`) from the model with the data for which the model was estimated.

Note the discrepancy between this value and the values shown during the minimization procedure (in `pem`, `armax`, `bj`, or `oe`), because these are the values of the *robustified* loss function (see `LimitError`). Note also that it is the nonrobustified residuals that are used to estimate the variance of *e* as stored in `Model.NoiseCovariance`. It is also this value that is used to estimate the covariance matrix of the estimated parameters. Outliers can thus influence the estimate of `NoiseVariance` and the covariance matrix, while the parameter estimates are made robust against them.

Be careful when comparing loss function values between different structures that use very different disturbance models. An output-error model might have

a better input-output fit even though it displays a higher value of the loss function than, say, an ARX model.

For ARX models computed using `iv4`, the covariance matrix of the innovations is estimated using the provisional disturbance model that is used to form the optimal instruments. The loss function therefore differs from what would be obtained if you computed the prediction errors using the model directly from the data. It is still the best available estimate of the innovations covariance. In particular, it is difficult to compare the loss function in an ARX model estimated using `arx` and one estimated using `iv4`.

Enumeration of Estimated Parameters

In some cases the parameters of a model are given just as an ordered list. This is the case for `m.ParameterVector` and also when online information from the minimization is displayed with `` `trace'='full'. ``

$$pars = [a_1, \ldots, a_{na}, b_1^1, \ldots, b_{nb1}^1, b_1^2, \ldots b_{nb2}^2, \ldots$$

$$b_1^{nu}, \ldots, b_{nbnu}^{nu}, c_1, \ldots, c_{nc}, d_1, \ldots, d_{nc},$$

$$f_1^1, \ldots f_{nf1}^1, \ldots, f_1^{nu}, \ldots, f_{nfnu}^{nu}]$$

Here the superscript refers to the input number:

- For a state-space structure defined by `idss`, the parameters in `m.ParameterValues` are obtained in the following order. The A matrix is first scanned row by row for free parameters. Then the B matrix is scanned row by row, and so on for the C, D, K, and $X0$ matrices.
- For a state-space matrix that is defined by `idgrey`, the ordering of the parameters is the same as in the user-written M-file.

Multivariable ARX models are internally represented in state-space form. The parameter ordering follows the one described above. The ordering of the parameters might not be transparent, however, so it is better to use `idarx` and `arxdata`.

Note that the property `PName` (for parameter name) might be useful to help with the bookkeeping in these cases, and when you are fixing certain parameters using `FixedParameter`. The routine `setpname` might be helpful in setting mnemonic parameter names automatically for black-box models.

Complex-Valued Data

Some applications of system identification work with complex-valued data, and thus create complex-valued models. Most of the routines in the System Identification Toolbox support complex data and models. This is true for the estimation routines `ar`, `armax`, `arx`, `bj`, `covf`, `ivar`, `iv4`, `oe`, `pem`, `spa`, and `n4sid`. The transformation routines, like `freqresp`, `zpkdata`, etc., also work for complex-valued models, but no pole-zero confidence regions are given. Note also that the parameter variance-covariance information then refers to the complex-valued parameters, so no separate information about the accuracy of the real and imaginary parts will be given. Some display functions like `compare` and `plot` do not work for the complex case. Use `sim` and plot real and imaginary parts separately.

Strange Results

Strange results can of course be obtained in any number of ways. We only point out two cautions: It is tempting in identification applications to call the residuals `eps`. *Don't do that*. This changes the machine ε, which certainly will give you strange results.

It is also natural to use names like `step`, `phase`, etc., for certain variables. Note, however, that these variables take precedence over M-files with the same names, so be sure you don't use variable names that are also names of M-files.

4

Function Reference

Functions — By Category (p. 4-2)
Functions — Alphabetical List (p. 4-11)

Functions — By Category

Help Functions

`advice`	Advice about data set or estimated model
`help ident`	List System Identification Toolbox commands
`idhelp`	Brief help for System Identification Toolbox commands
`idprops`, `help idprops`	List and explain the object properties

Graphical User Interface

`ident`	Open System Identification Toolbox GUI
`midprefs`	Set directory for storing idprefs.mat containing GUI startup information

Simulation and Prediction

`idinput`	Generate identification input signals
`idmdlsim`	Simulate `idmodel` objects in Simulink
`pe`	Compute prediction errors associated with model and data set
`predict`	Predict output *k* steps ahead
`sim`	Simulate linear models with confidence regions

Data Manipulation

`advice`	Advice about data set or estimated model
`delayest`	Estimate time delay (dead time) from data
`detrend`	Remove trends from output-input data
`diff`	Difference signals in `iddata` objects

`fcat`	Concatenate frequency-domain signals in `idfrd` and `iddata` objects
`feedback`	Investigate feedback presence in `iddata` sets
`fft/ifft`	Transform `iddata` objects between the time and the frequency domains
`fselect`	Select frequencies from `idfrd` object
`get`	Query `idmodel`, `idfrd`, and `iddata` properties
`getexp`	Retrieve experiment(s) from multiple-experiment `iddata` objects
`iddata`	Package input-output into `iddata` object
`idfilt`	Filter data using user-defined passbands, general filters, or Butterworth filters
`isreal`	Determine whether model or data set contains real parameters or data
`merge (iddata)`	Merge data sets into one `iddata` object
`misdata`	Reconstruct missing input and output data
`nkshift`	Shift data sequences
`nuderst`	Set step size for numerical differentiation
`pexcit`	Determine level of excitation of input signals
`plot (iddata)`	Plot input-output `iddata`
`realdata`	Determine whether `iddata` is based on real-valued signals
`resample`	Resample data by interpolation and decimation
`set`	Set properties of models and `iddata` sets

Nonparametric Estimation

`covf`	Estimate time-series covariance functions
`cra`	Prewhitened-based correlation analysis and impulse response

`delayest`	Estimate time delay (dead time) from data
`etfe`	Estimate empirical transfer functions and periodograms
`feedback`	Investigate feedback presence in `iddata` sets
`impulse`	Plot impulse response with confidence regions
`pexcit`	Determine level of excitation of input signals
`spa`	Estimate frequency response and spectrum using spectral analysis
`spafdr`	Estimate frequency response and spectrum using spectral analysis with frequency-dependent resolution
`step`	Plot step response with confidence regions

Parameter Estimation

`ar`	Estimate parameters of AR model for scalar time series
`armax`	Estimate parameters of ARMAX or ARMA model
`arx`	Estimate parameters of ARX or AR model using least squares
`bj`	Estimate parameters of Box-Jenkins model
`ivar`	Estimate AR model using instrumental variable methods
`iv4`	Estimate ARX model using four-stage instrumental variable method
`oe`	Estimate parameters of output-error model
`n4sid`	Estimate state-space model using subspace method
`pem`	Estimate parameters of general linear models

Model Structure Creation

`idarx`	Construct idarx model from ARX polynomials
`idfrd`	Construct `idfrd` object from `idmodel` object or functions
`idgrey`	Construct grey-box linear model using user-defined M-file
`idpoly`	Create structure for input-output models using numerator and denominator polynomials
`idproc`	Create simple, continuous-time process models
`idss`	Create structure for linear state-space models with known and unknown parameters

Manipulating Model Structures

`get`	Query `idmodel`, `idfrd`, and `iddata` properties
`init`	Set or randomize initial parameter values
`merge (idmodel)`	Merge estimated models
`selstruc`	Select model order (structure)
`set`	Set properties of models and `iddata` sets
`setstruc`	Set matrix structure for `idss` objects

Model Conversion

`arxdata`	ARX parameters with variance information from `idmodel` models
`balred`	Reduce model order (requires Control System Toolbox)
`c2d`	Convert model from continuous to discrete time
`d2c`	Convert model from discrete to continuous time
`frd`	Convert `idfrd` objects to freqency-response-data LTI models of Control System Toolbox
`freqresp`	Compute frequency function for model
`fselect`	Select frequencies from `idfrd` object
`idfrd`	Convert `idmodel` to `idfrd` object containing frequency functions and spectra
`noisecnv`	Convert `idmodel` with noise channels to model with only measured channels
`polydata`	Convert model to input-output polynomials
`ss`	Convert `idmodel` objects to state-space LTI models of Control System Toolbox
`ssdata`	Convert model to state-space form
`tf`	Convert `idmodel` objects to transfer-function LTI models of Control System Toolbox

`tfdata`	Convert model to transfer-function form
`zpk`	Convert `idmodel` objects to zero-pole-gain LTI models of Control System Toolbox
`zpkdata`	Compute zeros, poles, and transfer-function gains of models

Model Analysis

`advice`	Advice about the data set or estimated model
`bode`	Plot frequency functions in Bode diagram form with confidence regions
`compare`	Compare measured outputs with model outputs
`ffplot`	Plot frequency functions and spectra
`impulse`	Plot impulse response with confidence regions
`isreal`	Determine whether model or data set contains real parameters or data
`nyquist`	Plot Nyquist curve of frequency function with confidence regions
`present`	Display information in `idmodel` model, including uncertainty
`pzmap`	Plot zeros and poles with confidence regions
`step`	Plot step response with confidence regions
`view`	Plot model characteristics using LTI viewer in Control System Toolbox

Model Validation

`aic`	Akaike Information Criterion for estimated model
`arxstruc`	Compute loss function for set of different model structures of single-output ARX type
`compare`	Compare measured outputs with model outputs

`fpe`	Akaike Final Prediction Error for estimated model
`pe`	Compute prediction errors associated with model and data set
`predict`	Predict output *k* steps ahead
`resid`	Compute and test model residuals (prediction errors)
`selstruc`	Select model order (structure)
`sim`	Simulate linear models with confidence regions

Assessing Model Uncertainty

`arxdata`	ARX parameters with variance information from `idmodel` models
`bode`	Plot frequency functions in Bode diagram form with confidence regions
`impulse`	Plot impulse response with confidence regions
`nyquist`	Plot Nyquist curve of frequency function with confidence regions
`polydata`	Convert model to input-output polynomials
`pzmap`	Plot zeros and poles with confidence regions
`sim`	Simulate linear models with confidence regions
`simsd`	Simulate models with uncertainty using Monte Carlo method
`ssdata`	Convert model to state-space form
`step`	Plot step response with confidence regions
`tfdata`	Convert model to transfer-function form
`zpkdata`	Compute zero, poles, and transfer-function gains of models

Model Structure Selection

`arxstruc`	Compute loss function for set of different model structures of single-output ARX type
`ivstruc`	Compute loss functions for sets of output-error model structures
`n4sid`	Estimate state-space model using subspace method
`pem`	Estimate parameters of general linear models
`selstruc`	Select model order (structure)
`struc`	Generate model structure matrices

Recursive Parameter Estimation

`rarmax`	Estimate recursively parameters of ARMAX or ARMA model
`rarx`	Estimate recursively parameters of ARX or AR models
`rbj`	Estimate recursively parameters of Box-Jenkins model
`roe`	Estimate output-error models (IIR-filters) recursively
`rpem`	Estimate general input-output models using recursive prediction error method
`rplr`	Estimate general input-output models using recursive pseudolinear regression method
`segment`	Segment data and estimate models for each segment

General

`advice`	Advice about data set or estimated model
`get`	Query `idmodel`, `idfrd`, and `iddata` properties
`set`	Set properties of models and `iddata` sets
`setpname`	Set mnemonic parameter names for black-box model structures
`size`	Dimensions of `iddata`, `idmodel`, and `idfrd` objects
`timestamp`	Return date and time when object was created or last modified

Functions — Alphabetical List

For ease of use, most functions have several default arguments. The Syntax first lists the function with the necessary input arguments and then with all the possible input arguments. The functions can be used with any number of arguments between these extremes. The rule is that missing, trailing arguments are given default values, as defined in the manual. Default values are also obtained by entering the arguments as the empty matrix [ ].

MATLAB does not require that you specify all of the output arguments; those not specified are not returned. For functions with several output arguments in the System Identification Toolbox, missing arguments are, as a rule, not computed, in order to save time.

The following reference pages are listed in alphabetical order.

advice

Purpose

Advice about data set or estimated model

Syntax

```
advice(Model)
advice(Data)
```

Description

`Model` is any estimated `idmodel` (`idarx`, `idgrey`, `idpoly`, `idproc`, `idss`).

`Data` is any data set in the `iddata` format.

The command gives text information to the Command Window about the data set or the model. Typical advice given is

- For data sets,
  - The excitation level of the signals and what consequences this has for what model orders can be supported (see also `pexcit`)
  - Whether detrending should be applied
  - Presence of output feedback in the data, and its consequences (see also `feedback`)
- For models,
  - Whether the model appears to have captured the essential dynamics of the system, and/or the disturbance characteristics
  - Whether the model seems to be of unnecessarily high order
  - Whether significant feedback effects in the validation data can be detected

See Also

`feedback`, `pexcit`

Purpose

Akaike Information Criterion for estimated model

Syntax

```
am = aic(Model1,Model2,...)
```

Description

`Model` is any estimated `idmodel` (`idarx`, `idgrey`, `idpoly`, `idproc`, `idss`).

`am` is returned as a row vector with the values of Akaike's Information Criterion (AIC) for each of the models. The AIC is given as

$$AIC = \log(V) + \frac{2d}{N}$$

where V is the loss function, d is the number of estimated parameters, and N is the number of estimation data values.

Here

$$V = det\left(\frac{1}{N}\sum_{1}^{N}\varepsilon(t,\hat{\theta}_N)(\varepsilon(t,\hat{\theta}_N))^T\right) \quad \textbf{(4-1)}$$

where $\hat{\theta}_N$ is the parameter estimate.

The AIC is formally defined as the value of the negative log-likelihood function at the estimated parameters plus the number of estimated parameters. The connection between this and the expressions above is as follows (cf (7.92)ff in Ljung (1999)):

If the disturbance source is Gaussian with covariance matrix Λ, the logarithm of likelihood function is

$$L(\theta,\Lambda) = -\frac{1}{2}\sum_{1}^{N}\varepsilon(t,\theta)^T\Lambda^{-1}\varepsilon(t,\theta) - \frac{N}{2}\log det\Lambda + const$$

Maximizing this analytically with regard to Λ gives, and then maximizing the result with regard to θ, gives

$$L(\theta,\hat{\Lambda}) = const + \frac{Np}{2} + \frac{N}{2}\log(V)$$

where p is the number of outputs and V is defined by (Equation 4-1). After removing constants and suitable normalization, the desired expression is reached.

References

Sections 7.4 and 16.4 in Ljung (1999).

See Also

`EstimationInfo`, `fpe`

Purpose

Algorithm properties affecting estimation process

Syntax

```
idprops algorithm
m.algorithm
```

Description

All the `idmodel` objects in the toolbox, `idarx`, `idss`, `idpoly`, `idproc`, and `idgrey`, have a property `Algorithm`, which is a structure that contains a number of options that govern the estimation algorithms. The fields of this structure can be individually set and retrieved in the usual way, such as `get(m,'MaxIter')` or `m.SearchDirection = 'gn'`. Also, autofill applies and the names are case insensitive.

Note `Algorithm` is a property of `idmodel`. Any algorithm property can be separately set as above. Also, if you have a standard algorithm setup that you prefer, you can set those properties simultaneously, as in `m = pem(Data,mi,'alg',myalg)`.

Note The algorithm properties, like all other model properties, are inherited by the resulting model `m`. If you continue the estimation using `m` as the initial model, all previously set algorithm features will thus apply, unless you specify otherwise.

The fields of `Algorithm` are as follows:

Applying to All Estimation Methods

- `Focus`: This property affects the weighting applied to the fit between the model and the data. It can be used to assure that the model approximates the true system well over certain frequency intervals. `Focus` can assume the following values:
  - `'Prediction'`: This is the default and means that the model is determined by minimizing the prediction errors. It corresponds to a frequency weighting that is given by the input spectrum times the inverse noise model. Typically, this favors a good fit at high frequencies. From a

statistical variance point of view, this is the optimal weighting, but then the approximation aspects (bias) of the fit are neglected.

- `'Simulation'`: This means that frequency weighting of the transfer function fit is given by the input spectrum. Frequency ranges where the input has considerable power will thus be better described by the model. In other words, the model approximation is such that the model will produce as good simulations as possible, when applied to inputs with the same spectra as used for the estimation. For models that have no disturbance model, that is $y = G\,u + e$, (`A=C=D=1` for `idpoly` models and `K = 0` for `idss` models) there is no difference between `'Simulation'` and `'Prediction'`. For models with a disturbance description, that is, $y = Gu + H\,e$, G is first estimated with $H = 1$ and then H is estimated by a prediction error method, keeping the estimated transfer function $\hat{G}$ fixed. This option also guarantees a stable transfer function G.
- `'Stability'`: The resulting model is guaranteed to be stable, but a prediction weighing is still maintained. Note that forcing the model to be stable could mean that a bad model is obtained. Use only when you know the system to be stable.
- *A row vector or matrix defining passbands*:

  `[wl,wh] or [w1l,w1h;w2l,w2h;w3l,w3h;...]`

  where `wl` and `wh` define upper and lower limits for a passband. With several rows, the union of passbands defined be each row is obtained. The fit between data and model will the be focused on the passband(s) thus defined.
- *Any SISO linear filter*: The transfer function from input to output is determined by a frequency fit with this filter times the input spectrum as weighting function. The disturbance model is determined by a prediction error method, keeping the transfer function estimate fixed, as in the simulation case. To obtain a good model fit over a special frequency range, the filter should thus be chosen with a passband over this range. For a model with no disturbance model, the result is the same as first applying prefiltering to data using `idfilt`. The filter can be specified in a few different ways as

  Any single-input-single-output `idmodel`

  An `ss`, `tf`, or `zpk` model from the Control System Toolbox

  `{A,B,C,D}` with the state-space matrices for the filter

{numerator, denominator} with the transfer function numerator/denominator of the filter

- For frequency-domain data, 'Focus' can also be given as a column vector of weights. The vector must be of the same size as Data.Frequency. Each input and output response in the data is then multiplied by the corresponding weight at the respective frequencies.

- MaxSize: No matrix with more than MaxSize elements is formed by the algorithm, whenever possible. Instead, for loops are used. MaxSize thus decides the memory/speed tradeoff, and can prevent slow use of virtual memory. MaxSize can be any positive integer, but the input-output data must contain fewer than MaxSize elements. The default value of MaxSize is 'Auto', which means that the value is determined in the M-file idmsize. You can edit this file to optimize speed on a particular computer. Generally speaking, MaxSize does not affect the numerical properties of the estimate. The only exception is when you use InitialState = 'backcast' for frequency-domain data. Then the frequency ranges where the backcasting takes place may depend on MaxSize, resulting in slightly different estimates.
- FixedParameter: A list of parameters that will be kept fixed to the nominal/initial values and not estimated. This is a vector of integers containing the indices of the fixed parameters. The numbering of the parameters is the same as in the model property 'ParameterVector'. The parameter names from the property 'PName' can also be used. For structured state-space models, it is easier to fix/unfix parameters by the structure matrices As, Bs, etc. See idss. When you use parameter names to specify the fixed parameters, Fixedparameter is a cell array of strings. The strings can contain the wildcards '*' (meaning any continuation of the given string) and '?' (meaning any character). For example, if all disturbance model parameters start with 'k', FixedParameter = {'k*'} will fix all these parameters. The function setpname can be useful in this context.

Applying to n4sid, Estimating State-Space Models

These also apply to pem for estimating black-box state-space models, since these are initialized by the n4sid estimate.

- N4Weight: This property determines some weighting matrices used in the singular-value decomposition that is a central step in the algorithm. Two choices are offered: 'MOESP', which corresponds to the MOESP algorithm by Verhaegen, and 'CVA', which is the canonical variable algorithm by

Larimore. See the reference page for `n4sid`. The default value is `'N4Weight' = 'Auto'`, which gives an automatic choice between the two options.

- `N4Horizon`: Determines the prediction horizons forward and backward used by the algorithm. This is a row vector with three elements: `N4Horizon = [r sy su]`, where `r` is the maximum forward prediction horizon; that is, the algorithm uses up to `r` step-ahead predictors. `sy` is the number of past outputs, and `su` is the number of past inputs that are used for the predictions. For an exact definition of these integers, see pages 209 and 210 in Ljung (1999), where they are called *r*, *s1*, and *s2*. These numbers can have a substantial influence on the quality of the resulting model, and there are no simple rules for choosing them. Making `'N4Horizon'` a `k`-by-3 matrix means that each row of `'N4Horizon'` is tried, and the value that gives the best (prediction) fit to data is selected. (This option cannot be combined with selection of model order.) If you specify only one column in `'N4Horizon'`, the interpretation is `r=sy=su`. The default choice is `'N4Horizon' = 'Auto'`, which uses an Akaike Information Criterion (AIC) for the selection of `sy` and `su`.

Applying to Estimation Methods Using Iterative Search for Minimizing a Criterion, That Is, armax, bj, oe, and pem

- `Trace`: This property determines the information about the iterative search that is provided to the MATLAB Command Window.
  - `'Trace' = 'Off'`: No information is written to the screen.
  - `'Trace' = 'On'`: Information about criterion values and the search process is given for each iteration.
  - `'Trace' = 'Full'`: The current parameter values and the search direction are also given (except in the `'Free' SSParameterization` case for `idss` models).
- `LimitError`: This variable determines how the criterion is modified from quadratic to one that gives linear weight to large errors. Errors larger than `LimitError` times the estimated standard deviation will carry a linear weight in the criterions.The default value of `LimitError` is `1.6`. `LimitError = 0` disables the robustification and leads to a purely quadratic criterion. The standard deviation is estimated robustly as the median of the absolute deviations from the median, divided by `0.7`. (See Equations (15.9) and (15.10) in Ljung (1999).) When estimating with frequency-domain data, `LimitError` is set to zero.

- `MaxIter`: The maximum number of iterations performed during the search for minimum. The iterations stops when `MaxIter` is reached or some other stopping criterion is satisfied. The default value of `MaxIter` is `20`. Setting `MaxIter = 0` returns the result of the startup procedure. The actual number of used iterations is given by the property `EstimationInfo.Iterations`.
- `Tolerance`: Based on the Gauss-Newton vector computed at the current parameter value, an estimate is made of the expected improvement of the criterion at the next iteration. When this expected improvement is less than `Tolerance`, measured in percent, the iterations are stopped. Default value is `0.01`.
- `SearchDirection`: The direction along which a line search is performed to find a lower value of the criterion function. It may assume the following values:
  - `'gn'`: The Gauss-Newton direction (inverse of the Hessian times the gradient direction). If no improvement is found along this direction, the gradient direction is also tried.
  - `'gns'`: A regularized version of the Gauss-Newton direction. Eigenvalues less than `GnsPinvTol` (see "Advanced" below) of the Hessian are neglected, and the Gauss-Newton direction is computed in the remaining subspace.
  - 'gna': An adaptive version of `gns`, suggested by Wills and Ninness (IFAC World congress, Prague 2005). Eigenvalues less than `gamma*max(sv)` of the Hessian are neglected , where `sv` are the singular values of the Hessian. The Gauss-Newton direction is computed in the remaining subspace. `gamma` has the initial value `InitGnaTol` (see below) and is increased by a factor `LmStep` each time the search fails to find a lower value of the criterion in less than 5 bisections. It is decreased by a factor `2LmStep` each time a search is successful without any bisections.
  - `'lm'`: The Levenberg-Marquardt method is used. This means that the next parameter value is `-pinv(H+d*I)*grad` from the previous one, where `H` is the Hessian, `I` is the identity matrix, and `grad` is the gradient. `d` is a number that is increased until a lower value of the criterion is found.
  - `'Auto'`: A choice among the above is made in the algorithm. This is the default choice.
- `Advanced`: This is a structure that contains detailed algorithm choices that normally the user does not need to get involved in. For detailed explanations, you must examine the code. `'Advanced'` has the following fields:

- `Search`: Contains fields with relevance for the iterative search:
  a. `GnsPinvTol`: The tolerance for the pseudoinverse used to compute the `gns` direction. See above. Default is `10^-9`.
  b. `InitGnaTol`: The initial value of `gamma` in the `gna` search algorithm. Default is `InitGnaTol =10^-4`
  c. `LmStep`: The next value of `d` in the LM method is `lmstep` times the previous one. Default is `LmStep = 2`.
  d. `StepReduction`: In the line search used for directions other than LM, the step is reduced by the factor `StepReduction` in each try. Default is `StepReduction = 2`.
  e. `MaxBisection`: The maximum number of bisections used by the line search along the search direction. Default is `25`.
  f. `LmStartValue`: The starting value of `d` in the LM method. Default is `0.001`.
  g. `RelImprovement`: The iterations are stopped if the relative improvement of the criterion is less than `RelImprovement`. Default is `RelImprovement = 0`.
- `Threshold`: Contains fields with thresholds for several tests:
  a. `Sstability`: used for stability test of continuous-time models. Model is considered stable if its rightmost pole is to the left of `Sstability`. Default is `0`.
  b. `Zstability`: used for stability test of discrete-time models. Model is considered stable if all poles are within the distance `Zstability` from the origin. Default is `1.01`.
- `AutoInitialState`: When `InitialState = 'Auto'`, the state is estimated if the ratio of the prediction error norm with zero initial state to the norm with estimated initial state exceeds `AutoInitialState`. Default is `1.2`.

References

For the iterative minimization, see Dennis, J.E., Jr., and R.B. Schnabel, *Numerical Methods for Unconstrained Optimization and Nonlinear Equations*, Prentice Hall, Englewood Cliffs, N.J., 1983.

For a general reference to the identification algorithms, see Ljung (1999), Chapter 10.

See Also

`armax`, `bj`, `EstimationInfo`, `n4sid`, `oe`, `pem`

Purpose

Estimate parameters of AR model for scalar time series

Syntax

```
m = ar(y,n)
[m ,refl] = ar(y,n,approach,window)
[m,refl] = ar(y,n,approach,window,Prop1,Value1,Prop2,Value2,...)
```

Description

The parameters of the AR model structure

$$A(q)y(t) = e(t)$$

are estimated using variants of the least squares method.

The `iddata` object `y` contains the time-series data (just one output channel). The scalar `n` specifies the order of the model to be estimated (the number of A parameters in the AR model).

Note that the routine is for scalar time series only. For multivariate data use `arx`.

The estimate is returned in `m` and stored as an `idpoly` model. For the two lattice-based approaches, `'burg'` and `'gl'` (see below), the variable `refl` is returned, containing the reflection coefficients in the first row and the corresponding loss function values in the second. The first column is the zeroth-order model, so that the `(2,1)` element of `refl` is the norm of the time series itself.

Variable `approach` allows you to choose an algorithm from a group of several popular techniques for computing the least squares AR model. Available methods are as follows:

`approach = 'fb'`: The forward-backward approach. This is the default approach. The sum of a least squares criterion for a forward model and the analogous criterion for a time-reversed model is minimized.

`approach = 'ls'`: The least squares approach. The standard sum of squared forward prediction errors is minimized.

`approach = 'yw'`: The Yule-Walker approach. The Yule-Walker equations, formed from sample covariances, are solved.

`approach = 'burg'`: Burg's lattice-based method. The lattice filter equations are solved using the harmonic mean of forward and backward squared prediction errors.

`approach = 'gl'`: A geometric lattice approach. As in Burg's method, but the geometric mean is used instead of the harmonic one.

Windowing, within the context of AR modeling, is a technique for dealing with the fact that information about past and future data is lacking. There are a number of variants available:

`window = 'now'`: No windowing. This is the default value, except when `approach = 'yw'`. Only actually measured data are used to form the regression vectors. The summation in the criteria starts only at time `n`.

`window = 'prw'`: Prewindowing. Missing past data are replaced by zeros, so that the summation in the criteria can be started at time zero.

`window = 'pow'`: Postwindowing. Missing end data are replaced by zeros, so that the summation can be extended to time `N + n` (`N` being the number of observations).

`window = 'ppw'`: Pre- and postwindowing. This is used in the Yule-Walker approach.

The combinations of approaches and windowing have a variety of names. The least squares approach with no windowing is also known as the *covariance method*. This is the same method that is used in the `arx` routine. The MATLAB default method, forward-backward with no windowing, is often called the *modified covariance method*. The Yule-Walker approach, least squares plus pre- and postwindowing, is also known as the *correlation method*.

Possible property name/property value pairs are

- `'MaxSize'`/Integer. See `Algorithm Properties` for an explanation of `maxsize`.
- `'Ts'`/Real positive number. Setting the sampling time (overriding the sampling time of `y`.
- `'Covariance'`/`'None'`: Suppressing the calculation of the covariance matrix.

Examples

Compare the spectral estimates of Burg's method with those found from the forward-backward nonwindowed method, given a sinusoid in noise signal.

```
y = sin([1:300]') + 0.5*randn(300,1);
y = iddata(y);
mb = ar(y,4,'burg');
```

```
mfb = ar(y,4);
bode(mb,mfb)
```

References

Marple, Jr., S.L., *Digital Spectral Analysis with Applications*, Prentice Hall, Englewood Cliffs, 1987, Chapter 8.

See Also

`arx`, `etfe`, `ivar`, `spa`

Purpose

Estimate parameters of ARMAX or ARMA model

Syntax

```
m = armax(data,orders)
m = armax(data,'na',na,'nb',nb,'nc',nc,'nk',nk)
m = armax(data,orders,'Property1',Value1,...,'PropertyN',ValueN)
```

Description

`armax` returns `m` as an `idpoly` object with the resulting parameter estimates, together with estimated covariances.

`armax` estimates the parameters of the ARMAX model structure

$$A(q)y(t) = B(q)u(t-nk) + C(q)e(t)$$

using a prediction error method.

`data` is an `iddata` object containing the output-input data. Only time domain data are supported by `armax`. Use `oe` for frequency-domain data instead. The model orders can be specified as `(...,'na',na,'nb',nb,...)` or by setting the argument `orders` to

```
orders = [na nb nc nk]
```

The parameters `na`, `nb`, and `nc` are the orders of the ARMAX model, and `nk` is the delay. Specifically,

$$na: \quad A(q) = 1 + a_1 q^{-1} + \ldots + a_{na} q^{-na}$$

$$nb: \quad B(q) = b_1 + b_2 q^{-1} + \ldots + b_{nb} q^{-nb+1}$$

$$nc: \quad C(q) = 1 + c_1 q^{-1} + \ldots + c_{nc} q^{-nc}$$

Alternatively, you can specify the vector as

```
orders = mi
```

where `mi` is an initial guess at the ARMAX model given in `idpoly` format. See "Polynomial Representation of Transfer Functions" on page 3-11 for more information.

For multiinput systems, `nb` and `nk` are row vectors, such that the `k`th entry corresponds to the order and delay associated with the `k`th input.

If `data` has no input channels and just one output channel (that is, it is a time series), then

```
orders = [na nc]
```

and `armax` calculates an ARMA model for the time series

$$A(q)y(t) = C(q)e(t)$$

The structure and the estimation algorithm are affected by any property name/property value pairs that are set in the input argument list. Useful properties are `'Focus'`, `'InitialState'`, `'Trace'`, `'MaxIter'`, `'Tolerance'`, `'LimitError'`, and `'FixedParameter'`.

See `Algorithm` `Properties`, `idpoly`, and `idmodel` for details of these properties and their possible values.

`armax` does not support multioutput models. Use the state-space model for this case (see `n4sid` and `pem`).

Algorithm

A robustified quadratic prediction error criterion is minimized using an iterative search algorithm, whose details are governed by the properties `'SearchDirection'`, `'MaxIter'`, `'Tolerance'`, and `'Advanced'`. The iterations are terminated when `MaxIter` is reached, when the expected improvement is less than `Tolerance`, or when a lower value of the criterion cannot be found. Information about the search is contained in `m.EstimationInfo`.

The initial parameter values for the iterative search, if not specified in `orders`, are constructed in a special four-stage LS-IV algorithm.

The cutoff value for the robustification is based on the property `LimitError` as well as on the estimated standard deviation of the residuals from the initial parameter estimate. It is not recalculated during the minimization.

A stability test of the predictor is performed to ensure that only models corresponding to stable predictors are tested. Generally, both $C(q)$ and $F_i(q)$ (if applicable) must have all their zeros inside the unit circle.

Information about the minimization is furnished to the screen in case the property `'Trace'` is set to `'On'` or `'Full'`. With `'Trace' = 'Full'`, current and previous parameter estimates (in column vector form, listing parameters in alphabetical order) as well as the values of the criterion function are given. The

Gauss-Newton vector and its norm are also displayed. With `'Trace' = 'On'` just criterion values are displayed.

References

Ljung (1999), Section 10.2.

See Also

`arx`, `bj`, `idmodel`, `idpoly`, `oe`, `pem`, `Algorithm Properties`, `EstimationInfo`

Purpose

Estimate parameters of ARX or AR model using least squares

Syntax

```
m = arx(data,orders)
m = arx(data,'na',na,'nb',nb,'nk',nk)
m= arx(data,orders,'Property1',Value1,...,'PropertyN',ValueN)
```

Description

The parameters of the ARX model structure

$$A(q)y(t) = B(q)u(t-nk) + e(t)$$

are estimated using the least squares method.

`data` is an `iddata` object that contains the output-input data. Both time and frequency-domain signals are supported, and `data` can also be a `frd` or `idfrd` frequency-response data object. However, multioutput continuous-time models are not supported by `arx`.

`orders` is given as

```
orders = [na nb nk]
```

defining the orders and delay of the ARX model. Specifically, in discrete time

$$na: \quad A(q) = 1 + a_1 q^{-1} + \ldots + a_{na} q^{-na}$$

$$nb: \quad B(q) = b_1 + b_2 q^{-1} + \ldots + b_{nb} q^{-nb+1}$$

See “Polynomial Representation of Transfer Functions” on page 3-11 for more information. The model orders can also be defined by explicit pairs `(...,'na',na,'nb',nb,'nk',nk,...)`.

`m` is returned as the least squares estimates of the parameters. For single-output data this is an `idpoly` object, otherwise an `idarx` object.

For a time series, `data` contains no input channels and `orders = na`. Then an AR model of order `na` for `y` is computed.

$$A(q)y(t) = e(t)$$

Models with several inputs

$$A(q)y(t) = B_1(q)u_1(t-nk_1) + \ldots B_{nu}(q)u_{nu}(t-nk_{nu}) + e(t)$$

are handled by allowing `nb` and `nk` to be row vectors defining the orders and delays associated with each input.

Multioutput Models

Models with several inputs and several outputs are handled by allowing `na`, `nb`, and `nk` to contain one row for each output number. See "Multivariable ARX Models: the idarx Model" on page 3-43 for exact definitions. In the multioutput case, `arx` minimizes the trace of the prediction error covariance matrix, that is, the norm

$$\sum_{t=1}^{N} e^T(t)e(t)$$

This can be changed to an arbitrary quadratic norm

$$\sum_{t=1}^{N} e^T(t)\Lambda^{-1}e(t)$$

with a weighting matrix `Lambda`, by

```
m = arx(data,orders,'NoiseVariance', Lambda)
```

In general `arx` can be called with another ARX model `m_initial` as an argument.

```
m = arx(data,m_initial)
```

Then the orders and the weighting matrix for the prediction errors are taken from `m_initial`. You can further modify `m_initial` by adding a list of property name/property value pairs to the arguments. This is especially useful if some parameters should be fixed by `'FixedParameter'`.

Continuous-Time Models

For models with one output, continuous-time models can be estimated from continuous-time (frequency-domain) data. The orders are then interpreted as `na` being the number of estimated denominator coeffcients and `nb` being the number of estimated numerator coefficients. This means that `na = 4`, `nb = 2` gives the model

$$G(s) = \frac{b_1 s + b_2}{s^4 + a_1 s^3 + a_2 s^2 + a_3 s + a_4}$$

For continuous-time models, the delay parameters `nk` have no meaning and should be omitted. Note that

- It is often useful to limit the fit to a smaller frequency range when using continuous-time data:

  ```
  m = arx(datac,[na nb],'focus',[0 wh])
  ```

- Estimating continuous-time ARX models often gives some bias. It might be better to use the `oe` method.

Further Options

The algorithm and model structure are affected by the property name/property value list in the input argument.

Useful options are reached by the properties `'Focus'`, `'InputDelay'`, `'FixedParameter'`, and `'MaxSize'`.

For time-domain data the signals are shifted, so that unmeasured signals are never required in the predictors. There is thus no need to estimate initial conditions in that case. For frequency-domain data, however, adjusting the data by "initial conditions" that support circular convolution may be necessary. See "Initial States for Frequency Domain Data" on page 3-101.

It is then helpful to use the property name/property value pair `'InitialState'/init`, where `init` is one of `'zero'`, `'estimate'`, or `'auto'`. The default is `'auto'`, which makes a data-dependent choice between `'zero'` (no adjustment) and `'estimate'`.

See `Algorithm Properties` for details of these properties and possible values.

When the true noise term $e(t)$ in the ARX model structure is not white noise and `na` is nonzero, the estimate does not give a correct model. It is then better to use `armax`, `bj`, `iv4`, or `oe`.

Algorithm

The least squares estimation problem is an overdetermined set of linear equations that is solved using QR factorization.

The regression matrix is formed so that only measured quantities are used (no fill-out with zeros). When the regression matrix is larger than `MaxSize`, the QR factorization is performed in a `for` loop.

Examples

Here is an example that generates data and estimates an ARX model.

```
A = [1  -1.5  0.7]; B = [0 1 0.5];
m0 = idpoly(A,B);
u = iddata([],idinput(300,'rbs'));
e = iddata([],randn(300,1));
y = sim(m0, [u e]);
z = [y,u];
m = arx(z,[2 2 1]);
```

See Also

`ar`, `ivx`, `iv4`, `Algorithm Properties`, `EstimationInfo`

Purpose

ARX parameters with variance information from `idmodel` models

Syntax

```
[A,B] = arxdata(m)
[A,B,dA,dB] = arxdata(m)
```

Description

`m` is the model as an `idarx` or `idpoly` model object. `arxdata` works on any `idarx` model. For `idpoly` it gives an error unless the underlying model is an ARX model, that is, the orders `nc=nd=nf=0`. (See the reference page for `idpoly`.)

`A` and `B` are returned in the standard multivariable ARX format (see `idarx`), describing the model.

$$y(t) + A_1 y(t-1) + A_2 y(t-2) + \ldots + A_{na} y(t-na) =$$

$$B_0 u(t) + B_1 u(t-1) + \ldots + B_{nb} u(t-nb) + e(t)$$

Here A_k and B_k are matrices of dimensions *ny*-by-*ny* and *ny*-by-*nu*, respectively. (*ny* is the number of outputs, that is, the dimension of the vector $y(t)$, and *nu* is the number of inputs.) See "Multivariable ARX Models: the idarx Model" on page 3-43.

The arguments `A` and `B` are 3-D arrays that contain the `A` matrices and the `B` matrices of the model in the following way:

`A` is an `ny-by-ny-by-(na+1)` array such that

```
A(:,:,k+1) = Ak
A(:,:,1) = eye(ny)
```

Similarly `B` is an `ny-by-nu-by-(nb+1)` array with

```
B(:,:,k+1) = Bk
```

Note that `A` always starts with the identity matrix, and that leading entries in `B` equal to zero means delays in the model. For a time series, `B = []`.

`dA` and `dB` are the estimated standard deviations of `A` and `B`.

See Also

`idarx`

Purpose

Compute loss functions for set of different model structures of single-output ARX type

Syntax

```
V = arxstruc(ze,zv,NN)
V = arxstruc(ze,zv,NN,maxsize)
```

Description

`NN` is a matrix that defines a number of different structures of the ARX type. Each row of `NN` is of the form

```
nn = [na nb nk]
```

with the same interpretation as described for `arx`. See `struc` for easy generation of typical `NN` matrices for single-input systems.

Each of `ze` and `zv` is an `iddata` object containing output-input data. Frequency-domain data and `idfrd` objects are also supported. Models for each of the model structures defined by `NN` are estimated using the data set `ze`. The loss functions (normalized sum of squared prediction errors) are then computed for these models when applied to the validation data set `zv`. The data sets `ze` and `zv` need not be of equal size. They could, however, be the same sets, in which case the computation is faster.

Note that `arxstruc` is intended for single-output systems only.

The output argument `V` is best analyzed using `selstruc`. It contains the loss functions in its first row. The remaining rows of `V` contain the transpose of `NN`, so that the orders and delays are given just below the corresponding loss functions. The last column of `V` contains the number of data points in `ze`. The selection of a suitable model structure based on the information in `v` is normally done using `selstruc`. See "Model Structure Selection and Validation" on page 3-70 for advice on model structure selection and cross validation.

See `Algorithm Properties` for an explanation of `maxsize`.

Examples

Compare first- to fifth-order models with one delay using cross validation on the second half of the data set. Then select the order that gives the best fit to the validation data set.

```
NN = struc(1:5,1:5,1);
V = arxstruc(z(1:200),z(201:400),NN);
nn = selstruc(V,0);
m = arx(z,nn);
```

See Also arx, ivstruc, n4sid, selstruc, struc

balred

Purpose

Reduce model order (requires Control System Toolbox)

Syntax

```
MRED = balred(M)
MRED = balred(M,ORDER,'DisturbanceModel','None')
```

Description

This function reduces the order of any model `M` given as an `idmodel` object. The resulting reduced-order model, `MRED`, is an `idss` model.

The function requires several routines in the Control System Toolbox.

`ORDER`: The desired order (dimension of the state-space representation). If `ORDER = []`, which is the default, a plot shows how the diagonal elements of the observability and controllability Gramians of a balanced realization decay with the order of the representation. You are then prompted to select an order based on this plot. The idea is that such a small element has a negligible influence on the input-output behavior of the model. We recommend that you choose an order such that only large elements in these matrices are retained.

`'DisturbanceModel'`: If the property `DisturbanceModel` is set to `'None'`, then an output-error model `MRED` is produced: that is, one with the Kalman gain equal to zero (see Equation 3-23 in "Chapter 3, "Tutorial"). Otherwise (default), the disturbance model is also reduced.

The function recognizes whether `M` is a continuous- or discrete-time model and performs the reduction accordingly. The resulting model, `MRED`, is similar to `M` in this respect.

There are several options for how the reduction is performed: `AbsTol, RelTol, Offset, Elimination`.

Algorithm

The function `balred` from the Control System Toolbox is used. The plot, in case `ORDER = []`, shows the vector `g` returned by `balreal`.

Examples

Build a high-order multivariable ARX model, reduce its order to `3`, and compare the frequency responses of the original and reduced models:

```
M = arx(data,[4*ones(3,3),4*ones(3,2),ones(3,2)]);
MRED = balred(M,3);
bode(M,MRED)
```

Use the reduced-order model as an initial condition for a third-order state-space model.

```
M2 = pem(data,MRED);
```

See Also

balreal

bj

Purpose

Estimate parameters of Box-Jenkins model

Syntax

```
m = bj(data,orders)
m = bj(data,'nb',nb,'nc',nc,'nd',nd,'nf',nf,'nk',nk)
m = bj(data,orders,'Property1',Value1,'Property2',Value2,...)
```

Description

`bj` returns `m` as an `idpoly` object with the resulting parameter estimates, together with estimated covariances. The `bj` function estimates parameters of the Box-Jenkins model structure

$$y(t) = \frac{B(q)}{F(q)}u(t-nk) + \frac{C(q)}{D(q)}e(t)$$

using a prediction error method.

`data` is an `iddata` object containing the output-input data. Frequency-domain signals are not supported by `bj`. Use `oe` instead.

The model orders can be specified by setting the argument `orders` to

```
orders = [ nb nc nd nf nk]
```

The parameters `nb`, `nc`, `nd`, and `nf` are the orders of the Box-Jenkins model and `nk` is the delay. Specifically,

nf: $F(q) = 1 + f_1 q^{-1} + \ldots + f_{nf} q^{-nf}$

nb: $B(q) = b_1 + b_2 q^{-1} + \ldots + b_{nb} q^{-nb+1}$

nc: $C(q) = 1 + c_1 q^{-1} + \ldots + c_{nc} q^{-nc}$

nd: $D(q) = 1 + d_1 q^{-1} + \ldots + d_{nd} q^{-nd}$

The orders can also be defined as property name/property value pairs `(...,'nb',nb,...)`. Alternatively, you can specify the vector as

```
orders = mi
```

where `mi` is an initial guess at the Box-Jenkins model given in `idpoly` format. See “Polynomial Representation of Transfer Functions” on page 3-11 for more information.

For multiinput systems, nb, nf, and nk are row vectors with as many entries as there are input channels. Entry number i then describes the orders and delays associated with the ith input.

The structure and the estimation algorithm are affected by any property name/property value pairs that are set in the input argument list. Useful properties are 'Focus', 'InitialState', 'Trace', 'MaxIter', 'Tolerance', 'LimitError', and 'FixedParameter'.

See Algorithm Properties and the reference pages for idmodel and idpoly for details of these properties and their possible values.

bj does not support multioutput models. Use a state-space model for this case (see n4sid and pem).

Examples

Here is an example that generates data and stores the results of the startup procedure separately.

```
B = [0 1 0.5];
C = [1 -1 0.2];
D = [1 1.5 0.7];
F = [1 -1.5 0.7];
m0 = idpoly(1,B,C,D,F,0.1);
e = iddata([],randn(200,1));
u = iddata([],idinput(200));
y = sim(m0,[u e]);
z = [y u];
mi = bj(z,[2 2 2 2 1],'MaxIter',0)
m = bj(z,mi,'Maxi',10)
m.EstimationInfo
m = bj(z,m); % Continue if m.es.WhyStop shows that maxiter has
             % been reached.
compare(z,m,mi)
```

Algorithm

bj uses essentially the same algorithm as armax with modifications to the computation of prediction errors and gradients.

See Also

armax, idmodel, idpoly, oe, pem

bode

Purpose

Plot frequency functions in Bode diagram form with confidence regions

Syntax

```
bode(m)
[mag,phase,w] = bode(m)
[mag,phase,w,sdmag,sdphase] = bode(m)
bode(m1,m2,m3,...,w)
bode(m1,'PlotStyle1',m2,'PlotStyle2',...)
bode(m1,m2,m3,..'sd',sd,'mode',mode,'ap',ap)
bode(m1,m2,m3,'sd',sd,'mode',mode,'ap',ap,'fill')
```

Description

`bode` computes the magnitude and phase of the frequency response of `idmodel` and `idfrd` models. When invoked without left-hand arguments, `bode` produces a Bode plot on the screen.

`bode(m)` plots the Bode response of an arbitrary `idmodel` or `idfrd` model `m`. This model can be continuous or discrete, and SISO or MIMO. The `InputNames` and `OuputNames` properties of the models are used to plot the responses for different I/O channels in separate plots. Pressing the **Enter** key advances the plot from one input-output pair to the next one. Typing **Ctrl+C** aborts the plotting in an orderly fashion

If `m` contains information about both I/O channels and output noise spectra, only the I/O channels are shown. To show the output noise spectra, enter `m('n')` (`'n'` for `'noise'`) in the model list. Analogously, you can select specific I/O channels with normal subreferencing `m(ky,ku)`.

Argument w

`bode(m,w)` explicitly specifies the frequency range or frequency points to be used for the plot or for computing the response.

To focus on a particular frequency interval `[wmin,wmax]`, set `w = {wmin,wmax}` (notice the curly brackets). This plots the response for 100 frequency points logarithmically spaced from `wmin` to `wmax`. You can change this to `NP` points by using `w = {wmin,wmax,NP}`.

To use particular frequency points, set `w` to the vector of desired frequencies. Use `logspace` to generate logarithmically spaced frequency vectors. All frequencies should be specified in rad/s.

Note that the frequencies cannot be specified for `idfrd` objects. For those the plot and response are calculated for the internally stored frequencies. However, the plot is restricted to the range `{wmin,wmax}` if this is specified.

If no frequency range is specified, a default choice is made based on the dynamics of the model.

Property Name/Property Value Pairs 'sd'/sd, 'ap'/ap, and 'mode'/mode

The pairs can appear in any order or be omitted.

- `sd`: If `sd` is specified as a number larger than zero, confidence intervals for the functions are added to the graph as dash-dotted curves (of the same color as the estimate curve). They indicate the confidence regions corresponding to `sd` standard deviations. If an argument `'fill'` is included in the argument list, the confidence region is marked as a filled band instead.
- `ap`: By default, amplitude and phase plots are shown simultaneously for each I/O channel present in `m`. For spectra, phase plots are omitted. To show amplitude plots only, use `ap = 'A'`. For phase plots only, use `ap = 'P'`. The default is `ap = 'B'` for both plots.
- `mode`: To obtain all input/output plots in the same diagram use `mode = 'same'`.

Several Models

`bode(m1,m2,...,mN)` or `bode(m1,m2,...mN,w)` plots the Bode response of several `idmodel` or `idfrd` models on a single figure. The models can be mixes of different sizes and continuous/discrete. The sorting of the plots is based on the `InputNames` and `OutputNames`. If the frequencies `w` are specified, these will apply to all non-`idfrd` models in the list. If you want different frequencies for different models, you should thus first convert them to `idfrd` objects using the `idfrd` command.

`bode(m1,'PlotStyle1',...,mN,'PlotStyleN')` further specifies which color, line style, and/or marker should be used to plot each system, as in

```
bode(m1,'r--',m2,'gx')
```

Arguments

The output argument `w` contains the frequencies for which the response is given, whether specified among the input arguments or not. The output arguments `mag` and `phase` are 3-D arrays with dimensions

```
(number of outputs)x(number of inputs)x(length of w)
```

For SISO systems, `mag(1,1,k)` and `phase(1,1,k)` give the magnitude and phase (in degrees) at the frequency ω_k = `w(k)`. To obtain the result as a normal vector of responses, use `mag = mag(:)` and `phase = phase(:)`.

For MIMO systems, `mag(i,j,k)` is the magnitude of the frequency response at frequency `w(k)` from input `j` to output `i`, and similarly for `phase(i,j,k)`.

If `sdmag` and `sdphase` are specified, the standard deviations of the magnitude and phase are also computed. Then `sdmag` is an array of the same size as `mag`, containing the estimated standard deviations of the response, and analogously for `sdphase`.

See Also

`etfe`, `freqresp`, `idfrd`, `nyquist`, `spa`, `spafdr`

Purpose

Compare measured outputs with model outputs

Syntax

```
compare(data,m);
compare(data,m,k)
compare(data,m,k,'Samples',sampnr,'InitialState',init,'OutputPlots
   ',Yplots)
compare(data,m1,m2,...,mN)
compare(data,m1,'PlotStyle1',...,mN,'PlotStyleN')
[yh,fit,x0] = compare(data,m1,'PlotStyle1',...,mN,'PlotStyleN',k)
```

Description

`data` is the output-input data in the usual `iddata` object format. `data` can also be an `idfrd` object with frequency-response data.

`compare` computes the output `yh` that results when the model `m` is simulated with the input `u`. The result is plotted together with the corresponding measured output `y`. The percentage of the output variation that is explained by the model

```
fit = 100*(1 - norm(yh - y)/norm(y-mean(y)))
```

is also computed and displayed. For multioutput systems, this is done separately for each output. For frequency-domain data (or in general for complex valued data) the `fit` is still calculated as above, but only the absolute values of `y` and `yh` are plotted.

When the argument `k` is specified, the k step-ahead prediction of `y` according to the model `m` are computed instead of the simulated output. In the calculation of $yh(t)$, the model can use outputs up to time $t-k$: $y(s), s = t-k, t-k-1, \ldots$ (and inputs up to the current time t). The default value of `k` is `inf`, which gives a pure simulation from the input only. Note that for frequency-domain data, only simulation (`k = inf`) is allowed, and for time-series data (no input) only prediction (`k` not `inf`) is possible.

Property Name/Property Value Pairs

The optional property name/property value pairs `'Samples'/sampnr`, `'InitialState'/init`, and `'OutputPlots'/Yplots` can be given in any order.

The argument `Yplots` can be a cell array of strings. Only the outputs with `OutputName` in this array are plotted, while all are used for the necessary computations. If `Yplots` is not specified, all outputs are plotted.

The argument sampnr indicates that only the sample numbers in this row vector are plotted and used for the calculation of the fit. The whole data record is used for the simulation/prediction.

The argument init determines how to handle initial conditions in the models:

- init = 'e' (for 'estimate') estimates the initial conditions for best fit.
- init = 'm' (for 'model') used the model's internally stored initial state.
- init = 'z' (for 'zero') uses zero initial conditions.
- init = x0, where x0 is a column vector of the same size as the state vector of the models, uses x0 as the initial state.
- init = 'e' is the default.

Several Models

When several models are specified, as in compare(data,m1,m2,...,mN), the plots show responses and fits for all models. In that case data should contain all inputs and outputs that are required for the different models. However, some models might correspond to subselections of channels and might not need all channels in data. In that case the proper handling of signals is based on the InputNames and OutputNames of data and the models.

With compare(data,m1,'PlotStyle1',...mN,'PlotStyle2'), the color, line style, and/or marker can be specified for the curves associated with the different models. The markers are the same as for the regular plot command. For example,

```
compare(data,m1,'g_*',m2,'r:')
```

If data contains several experiments, separate plots are given for the different experiments. In this case sampnr, if specified, must be a cell array with as many entries as there are experiments.

Arguments

When output arguments [yh,fit,x0] = compare(data,m1,..,mN) are specified, no plots are produced.

yh is a cell array of length equal to the number of models. Each cell contains the corresponding model output as an iddata object.

fit is, in the general case, a 3-D array with fit(kexp,kmod,ky) containing the fit (computed as above) for output ky, model kmod, and experiment kexp.

`x0` is a cell array, such that `x0{kmod}` is the estimated initial state for model number `kmod`. If data is multiexperiment, `X0{kmod}` is a matrix whose column number `kexp` is the initial state vector for experiment number `kexp`.

Examples

Split the data record into two parts. Use the first one for estimating a model and the second one to check the model's ability to predict six steps ahead.

```
ze = z(1:250);
zv = z(251:500);
m= armax(ze,[2 3 1 0]);
compare(zv,m,6);
compare(zv,m,6,'Init','z') % No estimation of the initial state.
```

See Also

`sim`, `predict`

covf

Purpose

Estimate time-series covariance functions

Syntax

```
R = covf(data,M)
R = covf(data,M,maxsize)
```

Description

`data` is an `iddata` object and `M` is the maximum delay -1 for which the covariance function is estimated. The routine is intended for time-domain data only.

Let `z` contain the output and input channels

$$z(t) = \begin{bmatrix} y(t) \\ u(t) \end{bmatrix}$$

where y and u are the rows of `data.OutputData` and `data.InputData`, respectively, with a total of `nz` channels.

`R` is returned as an nz^2 -by- M matrix with entries

$$R(i+(j-1)nz,k+1) = \frac{1}{N}\sum_{t=1}^{N} z_i(t)z_j(t+k) = \hat{R}_{ij}(k)$$

where z_j is the `j`th row of z, and missing values in the sum are replaced by zero.

The optional argument `maxsize` controls the memory size as explained under `Algorithm Properties`.

The easiest way to describe and unpack the result is to use

```
reshape(R(:,k+1),nz,nz) = E z(t)*z'(t+k)
```

Here `'` is complex conjugate transpose, which also explains how complex data is handled. The expectation symbol `E` corresponds to the sample means.

Algorithm

When `nz` is at most two, and when permitted by `maxsize`, a fast Fourier transform technique is applied. Otherwise, straightforward summing is used.

See Also

`spa`

Purpose

Prewhitened-based correlation analysis and impulse response

Syntax

```
cra(data);
[ir,R,cl] = cra(data,M,na,plot);
cra(R);
```

Description

`data` is the output-input data given as an `iddata` object. The routine is intended for time-domain data only.

The routine only handles single-input-single-output data pairs. (For the multivariate case, apply `cra` to two signals at a time, or use `impulse`.) `cra` prewhitens the input sequence; that is, `cra` filters `u` through a filter chosen so that the result is as uncorrelated (white) as possible. The output `y` is subjected to the same filter, and then the covariance functions of the filtered `y` and `u` are computed and graphed. The cross correlation function between (prewhitened) input and output is also computed and graphed. Positive values of the lag variable then correspond to an influence from `u` to later values of `y`. In other words, significant correlation for negative lags is an indication of feedback from `y` to `u` in the data.

A properly scaled version of this correlation function is also an estimate of the system's impulse response `ir`. This is also graphed along with 99% confidence levels. The output argument `ir` is this impulse response estimate, so that its first entry corresponds to lag zero. (Negative lags are excluded in `ir`.) In the plot, the impulse response is scaled so that it corresponds to an impulse of height $1/T$ and duration T, where T is the sampling interval of the data.

The output argument `R` contains the covariance/correlation information as follows:

- The first column of `R` contains the lag indices.
- The second column contains the covariance function of the (possibly filtered) output.
- The third column contains the covariance function of the (possibly prewhitened) input.
- The fourth column contains the correlation function. The plots can be redisplayed by `cra(R)`.

The output argument `cl` is the 99% confidence level for the impulse response estimate.

The optional argument M defines the number of lags for which the covariance/correlation functions are computed. These are from -M to M, so that the length of R is 2M+1. The impulse response is computed from 0 to M. The default value of M is 20.

For the prewhitening, the input is fitted to an AR model of order na. The third argument of cra can change this order from its default value na = 10. With na = 0 the covariance and correlation functions of the original data sequences are obtained.

plot: plot = 0 gives no plots. plot = 1 (the default) gives a plot of the estimated impulse response together with a 99% confidence region. plot = 2 gives a plot of all the covariance functions.

An often better alternative to cra is the functions impulse and step, which use a high-order FIR model to estimate the impulse response.

Examples

Compare a second-order ARX model's impulse response with the one obtained by correlation analysis.

```
ir = cra(z);
m = arx(z,[2 2 1]);
imp = [1;zeros(19,1)];
irth = sim(m,imp);
subplot(211)
plot([ir irth])
title('impulse responses')
subplot(212)
plot([cumsum(ir),cumsum(irth)])
title('step responses')
```

See Also

impulse, step

Purpose

Convert model from continuous to discrete time

Syntax

```
md = c2d(mc,T)
md = c2d(mc,T,method)
[md,G] = c2d(mc,T,method)
```

Description

`mc` is a continuous-time model such as any `idmodel` object (`idgrey`, `idproc`, `idpoly`, or `idss`). `md` is the model that is obtained when it is sampled with sampling interval `T`.

`method = 'zoh'` (default) makes the translation to discrete time under the assumption that the input is piecewise constant (zero-order hold).

`method = 'foh'` assumes the input to be piecewise linear between the sampling instants (first-order hold).

With the Control System Toolbox, methods `'tustin'`, `'prewarp'`, and `'matched'` are also supported. In these cases the covariance matrix is not transformed.

Note that the innovations variance λ of the continuous-time model is interpreted as the intensity of the spectral density of the noise spectrum. The noise variance in `md` is thus given as λ / T.

`idpoly` and `idss` models are returned in the same format. `idgrey` structures are preserved if their `CDMfile` property is equal to `'cd'`. Otherwise they are transformed to `idss` objects. `idproc` models are returned as `idgrey` objects.

For `idpoly` models, the covariance matrix is translated by the use of numerical derivatives. The step sizes used for the differentiation are given by the function `nuderst`. For `idss, idproc,` and `idgrey` models, the covariance matrix is not translated, but covariance information about the input-output properties is included in `md`. To inhibit the translation of covariance information (which may take some time), use `c2d(mc,T,'covariance','none')`.

The output argument `G` is a matrix that transforms the initial state `x0` of `mc` to the initial state of `md` as

```
X0d=G * [X0; u(0)],
```

where `u(0)` is the input at time 0. For `idproc` models, the state variables correspond to those of `idgrey(mc)`. For `idpoly` models, `G` is returned as the empty matrix.

Examples

Define a continuous-time system and study the poles and zeros of the sampled counterpart.

```
mc = idpoly(1,1,1,1,[1 1 0],'Ts',0);
md = c2d(mc,0.5);
pzmap(md)
```

See Also

d2c

Purpose

Estimate time delay (dead time) from data

Syntax

```
nk = delayest(Data)
nk = delayest(Data,na,nb,nkmin,nkmax,maxtest)
```

Description

`Data` is an `iddata` object containing the input-output data. It can also be an `idfrd` object defining frequency-response data. Only single-output data can be handled.

`nk` is returned as an integer or a row vector of integers, containing the estimated time delay in samples from the input(s) to the output in `Data`.

The estimate is based on a comparison of ARX models with different delays:

$$y(t) + a_1 y(t-1) + \ldots + a_{na} y(t-na) = \ldots$$
$$b_1 u(t-nk) + \ldots + b_{nb} u(t-nb-nk+1)$$

The integer `na` is the order of the A polynomial (default 2). `nb` is a row vector of length equal to the number of inputs, containing the order(s) of the B polynomial(s) (default all 2).

`nkmin` and `nkmax` are row vectors of the same length as the number of inputs, containing the smallest and largest delays to be tested. Defaults are `nkmin = 0` and `nkmax = nkmin+20`.

If `nb`, `nkmax`, and/or `nkmin` are entered as scalars in the multiinput case, all inputs will be assigned the same values.

`maxtest` is the largest number of tests allowed (default 10,000).

detrend

Purpose

Remove trends from output-input data

Syntax

```
zd = detrend(z)
zd = detrend(z,o,brkp)
```

Description

`z` is an `iddata` object containing the input-output data. `detrend` removes the trend from each signal and returns the result as an `iddata` object `zd`.

The default (`o = 0`) removes the zeroth order trends; that is, the sample means are subtracted. If `z` is a frequency-domain data object, the response at frequency `0` is then set to zero,

With `o = 1`, linear trends are removed after a least squares fit. With `brkp` not specified, one single line is subtracted from the entire data record. A continuous piecewise linear trend is subtracted if `brkp` contains breakpoints at sample numbers given in a row vector.

Note that `detrend` for `iddata` objects differs somewhat from `detrend` in the Signal Processing Toolbox.

Examples

Remove a V-shaped trend from the output with its peak at sample number 119, and remove the sample mean from the input.

```
zd1(:,1,[]) = detrend(z(:,1,[]),1,119);
zd2(:,[],1) = detrend(z(:,[],1));
zd = [zd1,zd2];
```

Purpose

Difference signals in `iddata` objects

Syntax

```
zdi = diff(z)
zdi = diff(z,n)
```

Description

`z` is a time-domain `iddata` object. `diff(z)` and `diff(z,n)` apply this command to each of the input/output signals in `z`.

d2c

Purpose

Convert model from discrete to continuous time

Syntax

```
mc = d2c(md)
mc = d2c(md,method)
mc = d2c(md,'CovarianceMatrix',cov,'InputDelay',inpd)
```

Description

The discrete-time model `md`, given as any `idmodel` object, is converted to a continuous-time counterpart `mc`. The covariance matrix of the parameters in the model is also translated using the Gauss approximation formula and numerical derivatives of the transformation. The step sizes in the numerical derivatives are determined by the function `nuderst`. To inhibit the translation of the covariance matrix and save time, enter among the input arguments `(...,'CovarianceMatrix','None',....)` (any abbreviations will do).

`method` is one of the input intersample behaviors `'zoh'` (zero-order hold) or `'foh'` (first-order hold). If `method` is not specified, the `InterSample` behavior of the data from which `md` was estimated is used.

With the Control System Toolbox, methods `'tustin'`, `'prewarp'`, and `'matched'` are also supported. In these cases no translation of the covariance matrix takes place.

If the discrete-time model contains pure time delays, that is, $nk > 1$, then these are first removed before the transformation is made. These delays are appended as pure time delay (dead time) to the continuous-time model as the property `InputDelay`. To have the time delay approximated by a finite-dimensional continuous system, enter among the input arguments `(...,'InputDelay',0,...)`.

If the noise variance is λ in `md`, and its sampling interval is T, then the continuous-time model has an indicated level of noise spectral density equal to $T\lambda$.

While `idpoly` and `idss` models are returned in the same format, `idarx` models are returned as `idss` models `mc`. The reason is that the transformation does not preserve the special structure of `idarx`. The `idgrey` structures are preserved if their `CDMfile` property is equal to `cd`. Otherwise they are transformed to `idss` objects.

Note The transformation from discrete to continuous time is not unique. `d2c` selects the continuous-time counterpart with the slowest time constants consistent with the discrete-time model. The lack of uniqueness also means that the transformation can be ill-conditioned or even singular. In particular, poles on the negative real axis, in the origin, or in the point 1, are likely to cause problems. Interpret the results with care.

Examples

Transform an identified model to continuous time and compare the frequency responses of the two models.

```
m = n4sid(data,3)
mc = d2c(m);
bode(m.mc,'sd',3)
```

Note that you can include the transformation to continuous time in the `n4sid` command by specifying the model to be continuous time.

```
mc = n4sid(data,3,'Ts',0)
```

References

See “Discrete- and Continuous-Time Models” on page 3-68 and “Spectrum Normalization and the Sampling Interval” on page 3-107.

See Also

`c2d`, `nuderst`

EstimationInfo

Purpose

Information about estimation process results

Syntax

```
m.EstimationInfo
m.es
m.es.DataLength, etc
```

Description

Any estimated model has the property `EstimationInfo`, which is a structure whose fields give information about the results of the estimation. Depending on whether it is an estimated parametric `idmodel` or an estimated frequency response `idfrd`, `EstimationInfo` will contain different fields.

idmodel Case

The model structure will contain the properties `ParameterVector`, `CovarianceMatrix`, and `NoiseVariance`, which are all calculated in the estimation process (see the reference page for `idmodel`). In addition, `EstimationInfo` contains the following fields:

- `Status`: Information whether the model has been estimated, or modified after being estimated.
- `Method`: Name of the estimation command that produced the model.
- `LossFcn`: Value of the identification criterion at the estimate. Normally equal to the determinant of the covariance matrix of the prediction errors, that is, the determinant of `NoiseVariance`. Note that the loss function for the minimization might be different due to `LimitError`. The value of the nonrobustified loss function is always stored in `LossFcn`.
- `FPE`: Akaike's Final Prediction Error, defined as `LossFcn *(1+d/N}/(1-d/N)`, where `d` is the number of estimated parameters and `N` is the length of the data record.
- `DataName`: Name of the data set from which the model was estimated. This is equal to the property `name` of the `iddata` object. If this was not defined, the name of the MATLAB `iddata` variable is used.
- `DataLength`: Length of the data record.
- `DataTs`: Sampling interval of the data.
- `DataDomain`: `'Time'` or `'Frequency'`, depending on the data domain.
- `DataInterSample`: Intersample behavior of the data from which the model was estimated. This equals the property `InterSample` of the `iddata` object. (See `iddata`.)

- `WhyStop`: For models that have been estimated by iterative search. The stopping rule that caused the iterations to terminate. Assumes values like `'MaxIter reached'`, `'No improvement possible along the search vector'`, or `'Near (local) minimum'`. The latter means that the expected improvement is less than `Tolerance` (see `Algorithm Properties`).
- `UpdateNorm`: Norm of the Gauss-Newton vector at the last iteration.
- `LastImprovement`: Relative improvement of the criterion value at the last iteration.
- `Iterations`: Number of iterations used in the search.
- `InitialState`: Option actually used when `Model.InitialState = 'auto'`.
- `N4Weight`: For `n4sid` estimates, or estimates that have been initialized by `n4sid`: the actual value of `N4Weight` used.
- `N4Horizon`: For `n4sid` estimates, or estimates that have been initialized by `n4sid`: the actual value of `N4Horizon` used. See `n4sid` and `Algorithm Properties`.

idfrd Case

If the `idfrd` model is obtained from an estimated parametric model,

```
g = idfrd(Model)
```

`g.EstimationInfo` is the same as `Model.EstimationInfo` as described above.

For an `idfrd` model that has been estimated from `etfe`, `spa`, or `spafdr`, `EstimationInfo` contains the following fields:

- `Status`: Whether the model is estimated or directly constructed.
- `Method`: `etfe`, `spa`, or `spafdr`
- `WindowSize`: Resolution parameter (or vector) used for the estimation
- `DataName`, `DataLength`, `DataTs`, `DataDomain`, `DataInterSample`: Properties of the estimation data as above.

See Also

`idfrd`, `idmodel`

etfe

Purpose

Estimate empirical transfer functions and periodograms

Syntax

```
g = etfe(data)
g = etfe(data,M,N)
```

Description

`etfe` estimates the transfer function `g` of the general linear model

$$y(t) = G(q)u(t) + v(t)$$

`data` contains the output-input data and is an `iddata` object (time or frequency domain).

`g` is given as an `idfrd` object with the estimate of $G(e^{i\omega})$ at the frequencies

```
w = [1:N]/N*pi/T
```

The default value of `N` is `128`.

In case `data` contains a time series (no input channels), `g` is returned as the periodogram of `y`.

When `M` is specified other than the default value `M = []`, a smoothing operation is performed on the raw spectral estimates. The effect of `M` is then similar to the effect of `M` in `spa`. This can be a useful alternative to `spa` for narrowband spectra and systems, which require large values of `M`.

When `etfe` is applied to time series, the corresponding spectral estimate is normalized in the way that is defined in "Spectrum Normalization and the Sampling Interval" on page 3-107. Note that this normalization might differ from the one used by `spectrum` in the Signal Processing Toolbox.

If the (input) data is marked as periodic (`data.Period = integer`) and contains an even number of periods, the response is computed at the frequencies `k*2*pi/period` for `k = 0` up to the Nyquist frequency.

Examples

Compare an empirical transfer function estimate to a smoothed spectral estimate.

```
ge = etfe(z);
gs = spa(z);
bode(ge,gs)
```

Generate a periodic input, simulate a system with it, and compare the frequency response of the estimated model with the true system at the excited frequency points.

```
m = idpoly([1 -1.5 0.7],[0 1 0.5]);
u = iddata([],idinput([50,1,10],'sine'));
u.Period = 50;
y = sim(m,u);
me = etfe([y u])
bode(me,'b*',m)
```

Algorithm

The empirical transfer function estimate is computed as the ratio of the output Fourier transform to the input Fourier transform, using `fft`. The periodogram is computed as the normalized absolute square of the Fourier transform of the time series.

You obtain the smoothed versions (`M` less than the length of `z`) by applying a Hamming window to the output fast Fourier transform (FFT) times the conjugate of the input FFT, and to the absolute square of the input FFT, respectively, and subsequently forming the ratio of the results. The length of this Hamming window is equal to the number of data points in `z` divided by `M`, plus one.

See Also

`spa`, `spafdr`

fcat

Purpose

Concatenate frequency-domain signals in `idfrd` and `iddata` objects

Syntax

```
Mc = fcat(M1,M2,...Mn)
```

Description

`M1`, `M2`, etc., are all `idfrd` objects or `iddata` frequency-domain objects.

`Mc` is the corresponding object obtained by concatenation of the responses at all the frequencies in `Mk`.

Note that for `iddata` objects, this is the same as vertical concatenation (`vertcat`).

```
Mc = [M1;M2;..;Mn].
```

See Also

`fselect`, `iddata`, `idfrd`

Purpose

Investigate feedback presence in `iddata` sets

Syntax

```
[fbck,fbck0,nudir] = feedback(Data)
```

Description

`Data` is an `iddata` set with *Ny* outputs and *Nu* inputs.

`fbck` is an *Ny*-by-*Nu* matrix indicating the feedback. The *ky,ku* entry is a measure of feedback from output *ky* to input *ku*. The value is a probability *P* in percent. Its interpretation is that if the hypothesis that there is no feedback from output *ky* to input *ku* were tested at the level *P*, it would have been rejected. An intuitive but technically incorrect way of thinking about this is to see *P* as "the probability of feedback." Often only values above 90% are taken as indications of feedback. When `fbck` is calculated, direct dependence at lag zero between $u(t)$ and $y(t)$ is not regarded as a feedback effect.

`fbck0`: Same as `fbck`, but direct dependence at lag 0 between $u(t)$ and $y(t)$ is viewed as feedback effect.

`nudir`: A vector containing those input numbers that appear to have a direct effect on some outputs, that is, no delay from input to output.

See Also

`advice. idmodel/feedback`

ffplot

Purpose

Plot frequency functions and spectra

Syntax

```
ffplot(m)
[mag,phase,w] = ffplot(m)
[mag,phase,w,sdmag,sdphase] = ffplot(m)
ffplot(m1,m2,m3,...,w)
ffplot(m1,'PlotStyle1',m2,'PlotStyle2',...)
ffplot(m1,m2,m3,..'sd',sd,'mode',mode,'ap',ap)
```

Description

This function has exactly the same syntax as `bode`. The only difference is that it gives graphs with linear frequency scales and Hz as the frequency unit.

See Also

`bode`, `nyquist`

Purpose

Transform iddata objects between time and frequency domains

Syntax

```
Datf = fft(Data),  dat = ifft(Datf)
Datf = fft(Data,N)
Datf = fft(Data,N,'complex')
```

Description

If Data is a time-domain iddata object with real-valued signals and with constant sampling interval Ts, Datf is returned as a frequency-domain iddata object with the frequency values equally distributed from frequency 0 to the Nyquist frequency. Whether the Nyquist frequency actually is included or not depends on the signal length (even or odd). Note that the FFTs are normalized by dividing each transform by the square root of the signal length. That is in order to preserve the signal power and noise level.

In the default case, the length of the transformation is determined by the signal length. A second argument N will force FFT transformations of length N, padding with zeros if the signals in Data are shorter and truncating otherwise. Thus the number of frequencies in the real signal case will be N/2 or (N+1)/2. If Data contains several experiments, N can be a row vector of corresponding length.

For real signals, the default is that Datf only contains nonnegative frequencies. For complex-valued signals, negative frequencies are also included. To enforce negative frequencies in the real case, add a last argument, 'Complex'.

ifft similarily transforms a frequency-domain iddata object to the time domain. It requires the frequencies on Datf to be equally spaced from frequency 0 to the Nyquist frequency. More exactly this means that if there are N frequencies in Datf and the time sampling interval is Ts, then

Datf.Frequency = [0:df:F], where F is pi/Ts if N is odd and F = pi/Ts*(1-1/N) if N is even.

See Also

iddata, iddata/complex, iddata/realdata

frd

Purpose

Convert idfrd objects to freqency-response-data LTI models of Control System Toolbox

Syntax

```
sys = frd(mod)
```

Description

`mod` is an `idfrd` object. `sys` is returned as an `frd` object.

The fields `Frequency`, `ResponseData`, `Units`, `Ts`, `InputDelay`, `InputName`, `OutputName` and `Notes` in `mod` are transferred to `sys`. The remaing fields (`SpectrumData`, `CovarianceData` and `NoiseCovariance`) are ignored. The command therefore cannot be applied to a time-series `idfrd` model object.

See Also

`ss`, `tf`, `zpk`

Purpose

Compute frequency function for model

Syntax

```
H = freqresp(m)
[H,w,covH] = freqresp(m,w)
```

Description

`m` is any `idmodel` or `idfrd` object.

`H = freqresp(m,w)` computes the frequency response `H` of the `idmodel` model `m` at the frequencies specified by the vector `w`. These frequencies should be real and in rad/s.

If `m` has `ny` outputs and `nu` inputs, and `w` contains `Nw` frequencies, the output `H` is an `ny`-by-`nu`-by-`Nw` array such that `H(:,:,k)` gives the complex-valued response at the frequency `w(k)`.

For a SISO model, `H(:)` to obtain a vector of the frequency response.

If `w` is not specified, a default choice is made based on the dynamics of the model.

Output Arguments

```
[H,w,covH] = freqresp(M,w)
```

also returns the frequencies `w` and the covariance `covH` of the response. `covH` is a 5-D array where `covH(ky,ku,k,:,:)` is the 2-by-2 covariance matrix of the response from input `ku` to output `ky` at frequency `w(k)`. The 1,1 element is the variance of the real part, the 2,2 element is the variance of the imaginary part, and the 1,2 and 2,1 elements are the covariance between the real and imaginary parts. `squeeze(covH(ky,ku,k,:,:))` gives the covariance matrix of the corresponding response.

If `m` is a time series (no input channels), `H` is returned as the (power) spectrum of the outputs, an `ny`-by-`ny`-by-`Nw` array. Hence `H(:,:,k)` is the spectrum matrix at frequency `w(k)`. The element `H(k1,k2,k)` is the cross spectrum between outputs `k1` and `k2` at frequency `w(k)`. When `k1 = k2`, this is the real-valued power spectrum of output `k1`.

`covH` is then the covariance of the estimated spectrum `H`, so that `covH(k1,k1,k)` is the variance of the power spectrum estimate of output `k1` at frequency `W(k)`. No information about the variance of the cross spectra is normally given; that is, `covH(k1,k2,k) = 0` for `k1` is not equal to `k2`.)

If the model `m` is not a time series, use `freqresp(m('n'))` to obtain the spectrum information of the noise (output disturbance) signals.

Note that `idfrd` computes the same information as `freqresp`, and stores it in the `idfrd` object.

See Also

`bode`, `idfrd`, `nyquist`

Purpose

Akaike Final Prediction Error for estimated model

Syntax

```
fp = fpe(Model1,Model2,Model3,...)
```

Description

`Model` is any estimated `idmodel` (`idarx`, `idgrey`, `idpoly`, `idproc`, `idss`).

`fp` is returned as a row vector containing the values of the Akaike Final Prediction Error (FPE) for the different models. This is defined as

$$FPE = V\frac{1+d/N}{1-d/N}$$

where V is the loss function, d is the number of estimated parameters, and N is the number of estimation data.

Note that it is technically possible for FPE to become negative, if the number of estimated parameters exceeds the number of data (which could happen for models with many outputs). This is an artifact where the assumption behind the derivation that d/N is small is not valid. In such a case, it is better to use AIC.

References

Sections 7.4 and 16.4 in Ljung (1999).

See Also

`EstimationInfo`, `aic`

fselect

Purpose

Select frequencies from `idfrd` object

Syntax

```
idfm = fselect(idf,index)
idfm = fselect(idf,Fmin,Fmax)
```

Description

`idf` is any `idfrd` object. `index` is a row vector of frequency indices, so that `idfm` is the `idfrd` object that contains the response at frequencies `idf.Frequency(Index)`.

If `Fmin` and `Fmax` are specified, `idfm` contains responses at frequencies between `Fmin` and `Fmax`.

Note that the operation is the same as `dat(index)` for an `iddata` object.

Examples

Select every fifth frequency:

```
idfm = fselect(idf,5:5:100)
```

Select the response in the third quadrant:

```
ph = angle(squeeze(idf.response));
idfm = fselect(idf,find(ph>-pi & ph <-pi/2))
```

See Also

`fcat`

Purpose

Query idmodel, idfrd, and iddata properties

Syntax

```
Value = get(m,'PropertyName')
get(m)
Struct = get(m)
```

Description

`value = get(m,'PropertyName')` returns the current value of the property `PropertyName` of the `iddata` set or `idfrd`, or `idmodel` (`idgrey`, `idarx`, `idpoly`, `idss`) `m`. The string `'PropertyName'` can be the full property name (for example, `'SSParameterization'`) or any unambiguous case-insensitive abbreviation (for example, `'ss'`). You can specify any generic `idmodel` property or any property specific to `idgrey`, `idarx`, etc. (see `iddata`, `idmodel`, `idgrey`, `idarx`, `idpoly`, `idss`, and `Algorithm Properties` for lists of properties that can be accessed directly).

`Struct = get(m)` converts the object `m` into a standard MATLAB structure with the property names as field names and the property values as field values.

Without a left-hand argument

```
get(m)
```

displays all properties of `m` and their values.

Remarks

An alternative to the syntax

```
Value = get(m,'PropertyName')
```

is the structure-like referencing

```
Value = m.PropertyName
```

See Also

`arxdata`, `iddata`, `idfrd`, `idmodel`, `polydata`, `set`, `ssdata`, `tfdata`, `zpkdata`, `Algorithm Properties`, `EstimationInfo`

getexp

Purpose

Retrieve experiment(s) from multiple-experiment `iddata` objects

Syntax

```
d1 = getexp(data,ExperimentNumber)
d1 = getexp(data,ExperimentName)
```

Description

`data` is an `iddata` object that contains several experiments. `d1` is another `iddata` object containing the indicated experiment(s). The reference can either be by `ExperimentNumber`, as in `d1 = getexp(data,3)` or `d1 = getexp(data,[4 2])`; or by `ExperimentName`, as in `d1 = getexp(data,'Period1')` or `d1 = getexp(data,{'Day1','Day3'})`.

See `merge (iddata)` and `iddata` for how to create multiple-experiment data objects.

You can also retrieve the experiments using a fourth subscript, as in `d1 = data(:,:,:,ExperimentNumber)`. Type `help iddata/subsref` for details on this.

Purpose

Construct idarx model from ARX polynomials

Syntax

```
m = idarx(A,B,Ts)
m = idarx(A,B,Ts,'Property1',Value1,...,,'PropertyN',ValueN)
```

Description

`idarx` creates an object containing parameters that describe the general multiinput, multioutput model structure of ARX type.

$$y(t) + A_1 y(t-1) + A_2 y(t-2) + \ldots + A_{na} y(t-na) =$$

$$B_0 u(t) + B_1 u(t-1) + \ldots + B_{nb} u(t-nb) + e(t)$$

Here A_k and B_k are matrices of dimensions *ny*-by-*ny* and *ny*-by-*nu*, respectively. (*ny* is the number of outputs, that is, the dimension of the vector $y(t)$, and *nu* is the number of inputs.) See "Multivariable ARX Models: the idarx Model" on page 3-43.

The arguments `A` and `B` are 3-D arrays that contain the `A` matrices and the `B` matrices of the model in the following way.

`A` is an `ny`-by-`ny`-by-`(na+1)` array such that

```
A(:,:,k+1) = Ak
A(:,:,1) = eye(ny)
```

Similarly `B` is an `ny`-by-`nu`-by-`(nb+1)` array with

```
B(:,:,k+1) = Bk
```

Note that `A` always starts with the identity matrix, and that delays in the model are defined by setting the corresponding leading entries in `B` to zero. For a multivariate time series take `B = []`.

The optional property `NoiseVariance` sets the covariance matrix of the driving noise source $e(t)$ in the model above. The default value is the identity matrix.

The argument `Ts` is the sampling interval. Note that continuous-time models (`Ts` = 0) are not supported.

The use of `idarx` is twofold. You can use it to create models that are simulated (using `sim`) or analyzed (using `bode`, `pzmap`, etc.). You can also use it to define initial value models that are further adjusted to data (using `arx`). The free parameters in the structure are consistent with the structure of `A` and `B`; that

is, leading zeros in the rows of B are regarded as fixed delays, and trailing zeros in A and B are regarded as a definition of lower-order polynomials. These zeros are fixed, while all other parameters are free.

For a model with one output, ARX models can be described both as idarx and idpoly models. The internal representation is different, however.

idarx Properties

- A, B: The A and B polynomials as 3-D arrays, described above
- dA, dB: The standard deviations of A and B. Same format as A and B. Cannot be set.
- na, nb, nk: The orders and delays of the model. na is an ny-by-ny matrix whose *i-j* entry is the order of the polynomial corresponding to the *i-j* entry of A. Similarly nb is an ny-by-nu matrix with the orders of B. nk is also an ny-by-nu matrix, whose *i-j* entry is the delay from input *j* to output *i*, that is, the number of leading zeros in the *i-j* entry of B.
- InitialState: This describes how the initial state (initial values in filtering, etc.) should be handled. For time-domain applications, this is typically handled by starting the filtering when all data are available. For frequency-domain data, though, this requires more attention. See "Initial States for Frequency Domain Data" on page 3-101. The possible values of InitialState are 'zero', 'estimate', and 'auto' (which makes a data-dependent choice between zero and estimate).

In addition to these properties, idarx objects also have all the properties of the idmodel object. See idmodel, Algorithm Properties, and EstimationInfo.

Note that you can set and retrieve all properties either with the set and get commands or by subscripts. Autofill applies to all properties and values, and they are case insensitive.

For a complete list of property values, use get(m). To see possible value assignments, use set(m). See also idprops idarx.

Examples

Simulate a second-order ARX model with one input and two outputs, and then estimate a model using the simulated data.

```
A = zeros(2,2,3);
B = zeros(2,1,3)
A(:,:,1) =eye(2);
A(:,:,2) = [-1.5 0.1;-0.2 1.5];
A(:,:,3) = [0.7 -0.3;0.1 0.7];
B(:,:,2) = [1;-1];
B(:,:,3) = [0.5;1.2];
m0 = idarx(A,B,1);
u = iddata([],idinput(300));
e = iddata([],randn(300,2));
y = sim(m0,[u e]);
m = arx([y u],[[2 2;2 2],[2;2],[1;1]]);
```

See Also

`arx`, `arxdata`, `idmodel`, `idpoly`

iddata

Purpose

Package input-output data into `iddata` object

Syntax

```
data = iddata(y,u)
data = iddata(y,u,Ts,'Property1',Value1,...,'PropertyN',ValueN)
data = iddata(y,u,'Frequency',W)
data = iddata(idfrd_object)
```

Description

`iddata` is the basic object for dealing with signals in the toolbox. It is used by most of the commands. It can handle both time- and frequency-domain data. Most estimation and simulation commands can be applied to `iddata` objects in a transparent manner, regardless of the signal domain.

Basic Use

Let `y` be a column vector or an `N`-by-`ny` matrix. The columns of `y` correspond to the different output channels. Similarly, `u` is a column vector or an `N`-by-`nu` matrix containing the signals of the input channels. For time-domain signals,

```
data = iddata(y,u,Ts)
```

creates an `iddata` object containing these output and input channels. `Ts` is the sampling interval. This construction is sufficient for most purposes. For frequency-domain data, the vector of frequencies `W` (length `N`) at which the signals are defined must be supplied.

```
data = iddata(y,u,'Frequency',W)
```

The data is then plotted by `plot(data)` (see `plot`), and portions of the data record are selected, as in `ze = data(1:300)` or `zv = data(501:700)`.

An `idfrd` object can be transformed to a frequency-domain `iddata` object by

```
data = iddata(idfrd_object)
```

See “Transformations” on page 4-78.

The signals in the output channels are retrieved by `data.OutputData`, or for short, `data.y`. Similarly the input signals are obtained by `data.InputData` or `data.u`.

For a time series (no input channels), use `data = iddata(y)`, or let `u = []`.

An `iddata` object can also contain just an input, if you let `y = []`.

The sampling interval can be changed by `set(data,'Ts',0.3)` or, more simply, by `data.Ts = 0.3`.

The input and output channels are given default names like `'y1'`, `'y2'`, `'u1'`,`'u2'`, etc. You can set the channel names by

```
set(data,'InputName',{'Voltage','Current'},'OutputName','Tempera
ture')
```

(two inputs and one output in this example), and these names will then follow the object and appear in all plots. The names are also inherited by models that are estimated from the data.

Similarly, you can specify channel units using the properties `'OutputUnit'` and `'InputUnit'`. These units, when specified, are used in plots.

The time points associated with the time-domain data samples are determined by the sampling interval `Ts` and the time of the first sample, `Tstart`.

```
data.Tstart = 24
```

The actual time point values are given by the property `'SamplingInstants'` as in

```
plot(data.sa,data.u)
```

for a plot of the input with correct time points. Autofill is used for all properties, and they are case insensitive. For frequency-domain data, the property `'Frequency'` picks out the frequency values.

```
plot(data.fre,abs(data.u))
```

Manipulating Channels

An easy way to set and retrieve channel properties is to use subscripting. The subscripts are defined as

```
data(Samples,Outputs,Inputs)
```

so `dat(:,3,:)` is the data object obtained from `dat` by keeping all input channels, but only output channel 3. (Trailing ":"s can be omitted, so `dat(:,3,:) = dat(:,3)`.)

The channels can also be retrieved by their names, so that

```
dat(:,{'speed','flow'},[])
```

is the data object where the indicated output channels have been selected and no input channels are selected.

Moreover,

```
dat1(101:200,[3 4],[1 3]) = dat2(1001:1100,[1 2],[6 7])
```

will change samples 101 to 200 of output channels 3 and 4 and input channels 1 and 3 in the `iddata` object `dat1` to the indicated values from `iddata` object `dat2`. The names and units of these channels are also changed accordingly.

To add new channels, use horizontal concatenation of `iddata` objects.

```
dat =[dat1, dat2];
```

(see "Horizontal Concatenation" on page 4-78) or add the data record directly. Thus

```
dat.u(:,5) = U
```

adds a fifth input to `dat`.

Nonequal Sampling for Time-Domain Data

The property `'SamplingInstants'` gives the sampling instants of the data points. It can always be retrieved by `get(dat,'SamplingInstants')` (or `dat.s`) and is then computed from `dat.Ts` and `dat.Tstart`. `'SamplingInstants'` can also be set to an arbitrary vector of the same length as the data, so that nonequal sampling can be handled. `Ts` is then automatically set to `[]`. Most of the estimation routines, though, do not handle unequally sampled data.

Multiple Experiments

The `iddata` object can also store data from separate experiments. The property `'ExperimentName'` is used to separate the experiments. The number of data as well as the sampling properties can vary from experiment to experiment, but the input and output channels must be the same. (Use `NaN` to fill possibly unmeasured channels in certain experiments.) The data records will be cell arrays, where the cells contain data from each experiment.

You can define multiple experiments directly by letting the `'y'` and `'u'` properties, as well as `'Ts'` and `'Tstart'`, be cell arrays. (For frequency-domain data, the frequency vector will be a cell array.)

It is normally easier to create multiple-experiment data by merging experiments, as in

```
dat = merge(dat1,dat2)
```

See the reference page for `merge (data)`. Storing multiple experiments as one `iddata` object can be very useful to handle experimental data that has been collected on different occasions, or when a data set has been split up to remove "bad" portions of the data. All the toolbox's routines accept multiple-experiment data.

Experiments can be retrieved by the command `getexp`. They can also be retrieved by subscripting with a fourth index: `dat(:,:,:,3)` is experiment #3, and `dat(:,:,:,{'Day1','Day4'})` retrieves the two experiments with the indicated names.

The subscripting can be combined: `dat(1:100,[2,3],[4:8],3)` gives the 100 first samples of output channels 2 and 3 and input channels 4 to 8 of experiment #3. It can also be used for subassignment:

```
dat(:,:,:,'Run4') = dat2
```

which adds the data in `dat2` as a new experiment with name `'Run4'`. See `iddemo` #8 for an illustration of how multiple experiments can be used.

iddata Properties

In the list below, `N` denotes the number of samples of the signals, `ny` the number of output channels, `nu` the number of input channels, and `Ne` the number of experiments.

- `Domain`: Assumes the value `'Time'` or `'Frequency'` and denotes whether the data are time-domain or frequency-domain data.
- `Name`: An optional name for the data set. An arbitrary string.
- `OutputData`, `InputData`: The data matrices `y` and `u`. In the single-experiment case, `y` is an `N`-by-`ny` matrix and `u` is an `N`-by-`nu` matrix. For multiple experiments, `y` and `u` are 1-by-`Ne` cell arrays, with each cell containing the data for the different experiments.
- `OutputName`, `InputName`: Cell arrays of length `ny`-by-1 and `nu`-by-1 containing the names of the output and input channels. If not specified, default names `{'y1';'y2';...}` and `{'u1';'u2';...}` are given.
- `OutputUnit`, `InputUnit`: Cell arrays of length `ny`-by-1 and `nu`-by-1 containing the units of the output and input channels.

- `TimeUnit`: The unit for the sampling instants.
- `Ts`: Sampling interval. A scalar. For multiple-experiment data, `Ts` is a 1-by-`Ne` cell array, with each cell containing the sampling interval of the corresponding experiment. For nonequally sampled data, `Ts = []`. For time-domain signals, `Ts` has to be positive. For frequency-domain data, `Ts = 0` indicates continuous-time data; that is, the inputs and outputs are interpreted as continuous-time Fourier transforms of the signals, given at the frequencies in the frequency vector. Note that `Ts` is essential also for frequency-domain data, for proper interpretation of how the Fourier transforms were computed: They are interpreted as discrete-time Fourier transforms (DTFT) with the indicated sampling interval.
- `Tstart`: (For time-domain data only.) The starting time of the data record. A scalar. For multiple-experiment data, `Tstart` is a 1-by-`Ne` cell array, with each cell containing the starting time for the corresponding experiment.
- `SamplingInstants`: (For time-domain data only.) The time values of the sample points. An `N`-by-1 vector. For multiple-experiment data, `SamplingInstants` is a 1-by-`Ne` cell array, with each cell containing the sampling instants of the corresponding experiment. For equally sampled data, `SamplingInstants` is generated from `Ts` and `Tstart`.
- `Frequency`: (For frequency-domain data only.) The vector of frequencies at which the signals' transforms are defined. This is a column vector the length of the number of values of `OutputData` and `InputData`. For multiple-experiment data, `Frequency` is a cell array containing the frequencies for each experiment.
- `Units`: (For frequency-domain data only). The unit in which the frequencies are measured: rad/s or Hz. For multiple-experiment data, `units` is a cell array denoting the unit for each experiment.
- `Period`: The period of the input. A `nu`-by-1 vector, where the `k`th entry contains the period of the `k`th input. `Period = inf` means nonperiodic data. For multiple-experiment data, `Period` is a 1-by-`Ne` cell array with each cell containing the period(s) for the input of the corresponding experiment.
- `InterSample`: Describes the intersample behavior of the input channels. An `nu`-by-1 cell array where the `(k,1)` element is `'zoh'`, `'foh'`, or `'bl'`, denoting that input number `k` is piecewise constant, piecewise linear, or band limited. For multiple-experiment data, `InterSample` is an `nu`-by-`Ne` cell array.
- `ExperimentName`: A string containing the name of the experiment. For multiple-experiment data, `ExperimentName` is a 1-by-`Ne` cell array with each

cell containing the name of the corresponding experiment. It can be freely set, and is given names `{'Exp1', 'Exp2',...}` by default.

- `Notes`: An arbitrary field to store extra information and notes about the object.
- `UserData`: An arbitrary field for any possible use.

Note that you can set or retrieve all properties either with the `set` and `get` commands or by subscripts. Autofill applies to all properties and values, and they are case insensitive. `'y'` and `'u'` can be used as short for `'OutputData'` and `'InputData'`. `'y'` and `'u'` can also replace `'Output'` and `'Input'` in the other properties.

```
data.y=randn(100,2)
data.una = 'Voltage'
set(data,'tim','minute')
p = data.per
```

For a complete list of property values, use `get(data)`. To see possible value assignments, use `set(data)`.

Subreferencing

The samples, outputs and input channels can be referenced according to

```
data(samples,outputs,inputs)
```

Use a colon (`:`) to denote all samples/channels and the empty matrix (`[]`) to denote no samples/channels. For frequency-domain data, `samples` corresponds to the frequency vector indices, so that

```
dat2 = datf([5:30])
```

picks out the data values at frequencies `W(5:30)`, where `W = datf.Frequency`.

The channels can be referenced by number or by name. For several names, you must use a cell array.

```
dat2 = dat(:,'y3',{'u1','u4'})
dat2 = dat(:,3,[1 4])
```

Logical expressions will also work.

```
dat3 = dat2(dat2.sa>1.27&dat2.sa<9.3)
```

will select the samples with time marks between 1.27 and 9.3.

Subreferencing with a fourth argument refers to the experiment.

```
data(samples,outputs,inputs,Experiment)
```

Any subreferenced variable can also be assigned.

```
data(:,:,:,'Exp3'.y = flow(1:700,:)

data(1:10,1,1) = dat1(101:110,2,3)
```

Horizontal Concatenation

`dat = [dat1,dat2,...,datN]`creates an `iddata` object `dat`, consisting of the input and output channels in `dat1,... datN`. Default channel names (`'u1'`, `'u2'`, `'y1'`, `'y2'`, etc.) are changed so that overlaps in names are avoided, and the new channels are added.

If `datk` contains channels with user-specified names that are already present in the channels of `Datj, j<k`, these new channels are ignored.

Vertical Concatenation

`dat = [dat1;dat2;... ;datN]` creates an `iddata` object `dat` whose signals are obtained by stacking those of `datk` on top of each other. That is,

```
dat.OutputData = [dat1.Ouputdata;dat2.OutputData; ...
datN.OutputData]
```

and similarly for the inputs. The `datk` objects must all have the same number of channels and experiments.

Transformations

The command `fft` transforms a time-domain data set to frequency domain. The command `ifft` transforms a frequency-domain data set (with certain requirements on the frequency vector) to time domain.

An `idfrd` (frequency-response data) object can be transformed to a frequency-domain `iddata` object by

```
datf = iddata(idfrdobj)
```

The command

```
datf = iddata(idfrdobj,'me')
```

transforms the `idfrd` object to a multiple-experiment data set `datf` where each experiment corresponds to each of the inputs in `idfrdobj`. By default this transformation strips away frequencies where the response is `inf` or `NaN`. To keep these, use `datf = iddata(idfrdobj,'inf')`. Type `help idfrd/iddata`.

Dealing with Complex-Valued Data

If `Dat` is complex valued, `abs(Dat)`, `real(Dat)`, `imag(Dat)`, `phase(Dat)`, `angle(Dat)`, and `(Dat)` create `iddata` objects where each of the signals has been subjected to the indicated operation.

`isreal(Dat)` returns 1 if `Dat` contains only real signals, while `realdata(Dat)` returns 1 if the underlying signal is real. Thus a frequency-domain signal `Datf` obtained by `fft` from a real-valued time-domain signal will have

```
isreal(Datf) = 0 and realdata(Datf) = 1
```

For a `realdata` frequency-domain set (which only stores the values for nonnegative frequencies), the command

```
datc = complex(dat)
```

adds signal values for negative frequencies (by complex conjugation).

Online Help Functions

See `help iddata`, `idprops iddata`, `help iddata/subsref`, `help iddata/subsasgn`, `help iddata/horzcat`, and `help iddata/vertcat`.

See Also

`plot (iddata)`, `size`, `fft`, `ifft`, `detrend`, `idfilt`

ident

Purpose

Open System Identification Toolbox GUI

Syntax

```
ident
ident(session,directory)
```

Description

`ident` by itself opens the main interface window, or brings it forward if it is already open.

`session` is the name of a previous session with the graphical user interface, and typically has extension `.sid`. The `directory` argument is the complete path for the location of this file. If the session file is on the `MATLABPATH`, `directory` can be omitted.

When the session is specified, the interface will open with this session active. Typing `ident(session,directory)` on the MATLAB command line, when the interface is active, will load and open the session in question.

For more information about the graphical user interface, see Chapter 2, "The Graphical User Interface."

Examples

```
ident('iddata1.sid')
ident('mydata.sid','\matlab\data\cdplayer\')
```

See Also

`midprefs`

Purpose

Filter data using user-defined passbands, general filters, or Butterworth filters

Syntax

```
Zf = idfilt(Z,filter)
Zf = idfilt(Z,filter,causality)
Zf = idfilt(Z,filter,'FilterOrder',NF)
```

Description

`Z` is the data, defined as an `iddata` object. `Zf` contains the filtered data as an `iddata` object. The filter can be defined in three ways:

- As an explicit system that defines the filter,

  ```
  filter = idm or filter = {num,den} or filter = {A,B,C,D}
  ```

  `idm` can be any SISO `idmodel` or LTI model object. Alternatively the filter can be defined as a cell array `{A,B,C,D}` of SISO state-space matrices or as a cell array `{num,den}` of numerator/denominator filter coefficients.
- As a vector or matrix that defines one or several passbands,

  ```
  filter=[[wp1l,wp1h];[ wp2l,wp2h]; ....;[wpnl,wpnh]]
  ```

  The matrix is `n`-by-2, where each row defines a passband in rad/s. A filter is constructed that gives the union of these passbands. For time-domain data, it is computed as cascaded Butterworth filters or order NF. The default value of NF is `5`.

  For example, to define a stopband between `ws1` and `ws2`, use

  ```
  filter = [0 ws1; ws2,Nyqf]
  ```

  where `Nyqf` is the Nyquist frequency.
- For frequency-domain data, only the frequency response of the filter can be specified:

  ```
  filter = Wf
  ```

  Here `Wf` is a vector of possibly complex values that define the filter's frequency response, so that the inputs and outputs at frequency `Z.Frequency(kf)` are multiplied by `Wf(kf)`. `Wf` is a column vector of length = number of frequencies in `Z`. If the data object has several experiments, `Wf` is a cell array of length = # of experiments in `Z`.

For time-domain data, the filtering is carried out in the time domain as causal filtering as default. This corresponds to a last argument `causality =`

'causal'. With causality = 'noncausal', a noncausal, zero-phase filter is used for the filtering (corresponding to filtfilt in the Signal Processing Toolbox).

For frequency-domain data, the signals are multiplied by the frequency response of the filter. With the filters defined as passband, this gives ideal, zero-phase filtering ("brickwall filters"). Frequencies that have been assigned zero weight by the filter (outside the passband, or via the frequency response) are removed from the iddata object Zf.

It is common practice in identification to select a frequency band where the fit between model and data is concentrated. Often this corresponds to bandpass filtering with a passband over the interesting breakpoints in a Bode diagram. For identification where a disturbance model is also estimated, it is better to achieve the desired estimation result by using the property 'Focus' (see Algorithm Properties) than just to prefilter the data. The proper values for 'Focus' are the same as the argument filter in idfilt.

Algorithm

The Butterworth filter is the same as butter in the Signal Processing Toolbox. Also, the zero-phase filter is equivalent to filtfilt in that toolbox.

References

Ljung (1999), Chapter 14.

See Also

Algorithm Properties, idresamp, detrend

Purpose

Construct `idfrd` object from `idmodel` object or functions

Syntax

```
h = idfrd(Response,Freqs,Ts)
h = idfrd(Response,Freqs,Ts,'CovarianceData',Covariance, ...
   'SpectrumData',Spec,'NoiseCovariance',Speccov,'property1', ...
    Value1,'PropertyN',ValueN)
h = idfrd(mod)
h = idfrd(mod,Freqs)
```

Description

`idfrd` creates the `idfrd` model object.

For a model

$$y(t) = G(q)u(t) + H(q)e(t)$$

stores the transfer function estimate G (see (Equation 3-4) in Chapter 3, "Tutorial,")

$$G(e^{i\omega})$$

as well as the spectrum of the additive noise (Φ_v) at the output

$$\Phi_v(\omega) = \lambda T\left|H(e^{i\omega T})\right|^2$$

where λ is the estimated variance of $e(t)$, and T is the sampling interval.

Creating idfrd from Given Responses

`Response` is a 3-D array of dimension `ny`-by-`nu`-by-`Nf`, with `ny` being the number of outputs, `nu` the number of inputs, and `Nf` the number of frequencies (that is, the length of `Freqs`). `Response(ky,ku,kf)` is thus the complex-valued frequency response from input `ku` to output `ky` at frequency ω `=Freqs(kf)`. When defining the response of a SISO system, `Response` can be given as a vector.

`Freqs` is a column vector of length `Nf` containing the frequencies of the response.

`Ts` is the sampling interval. `T = 0` means a continuous-time model.

`Covariance` is a 5-D array containing the covariance of the frequency response. It has dimension `ny`-by-`nu`-by-`Nf`-by-2-by-2. The structure is such that

`Covariance(ky,ku,kf,:,:)` is the 2-by-2 covariance matrix of the response `Response(ky,ku,kf)`. The 1-1 element is the variance of the real part, the 2-2 element is the variance of the imaginary part, and the 1-2 and 2-1 elements are the covariance between the real and imaginary parts. `squeeze(Covariance(ky,ku,kf,:,:))` thus gives the covariance matrix of the corresponding response.

The information about spectrum is optional. The format is as follows:

`spec` is a 3-D array of dimension `ny-by-ny-by-Nf`, such that `spec(ky1,ky2,kf)` is the cross spectrum between the noise at output `ky1` and the noise at output `ky2`, at frequency `Freqs(kf)`. When `ky1 = ky2` the (power) spectrum of the noise at output `ky1` is thus obtained. For a single-output model, `spec` can be given as a vector.

`speccov` is a 3-D array of dimension `ny-by-ny-by-Nf`, such that `speccov(ky1,ky1,kf)` is the variance of the corresponding power spectrum. Normally, no information is included about the covariance of the nondiagonal spectrum elements.

If only `SpectrumData` is to be packaged in the `idfrd` object, set `Response = []`.

Creating idfrd from a Given Model

`idfrd` can also be computed from a given model `mod` (defined as any `idmodel` object).

If the frequencies `Freqs` are not specified, a default choice is made based on the dynamics of the model `mod`.

If `mod` has `InputDelay` different from zero, these are appended as phase lags, and `h` will then have an `InputDelay` of `0`.

The estimated covariances are computed using the Gauss approximation formula from the uncertainty information in `mod`. For models with complicated parameter dependencies, numerical differentiation is applied. The step sizes for the numerical derivatives are determined by `nuderst`.

Frequency responses for submodels can be obtained by the standard subreferencing, `h = idfrd(m(2,3))`. See `idmodel`. In particular, `h = idfrf(m('measured'))` gives an `h` that just contains the `ResponseData (G)` and no spectra. Also `h = idfrd(m('noise'))` gives an `h` that just contains `SpectrumData`.

The idfrd models can be graphed with bode, ffplot, and nyquist, which all accept mixtures of idmodel and idfrd models as arguments. Note that spa, spafdr, and etfe return their estimation results as idfrd objects.

idfrd Properties

- ResponseData: 3-D array of the complex-valued frequency response as described above. For SISO systems use Response(1,1,:) to obtain a vector of the response data.
- Frequency: Column vector containing the frequencies aT which the responses are defined.
- CovarianceData: 5-D array of the covariance matrices of the response data as described above.
- SpectrumData: 3-D array containing power spectra and cross spectra of the output disturbances (noise) of the system.
- NoiseCovariance: 3-D array containing the variances of the power spectra, as explained above.
- Units: Unit of the frequency vector. Can assume the values 'rad/s' and 'Hz'.
- Ts: Scalar denoting the sampling interval of the model whose frequency response is stored. 'Ts' = 0 means a continuous-time model.
- Name: An optional name for the object.
- InputName: String or cell array containing the names of the input channels. It has as many entries as there are input channels.
- OutputName: Correspondingly for the output channels.
- InputUnit: Units in which the input channels are measured. It has the same format as 'InputName'.
- OutputUnit: Correspondingly for the output channels.
- InputDelay: Row vector of length equal to the number of input channels. Contains the delays from the input channels. These should thus be appended as phase lags when the response is calculated. This is done automatically by freqresp, bode, ffplot, and nyquist. Note that if the idfrd is calculated from an idmodel, possible input delays in that model are converted to phase lags, and the InputDelay of the idfrd model is set to zero.
- Notes: An arbitrary field to store extra information and notes about the object.
- UserData: An arbitrary field for any possible use.

- `EstimationInfo`: Structure that contains information about the estimation process that is behind the frequency data. It contains the following fields (see also the reference page for `EstimationInfo`).
  - `Status`: Gives the status of the model, for example, `'Not estimated'`.
  - `Method`: The identification routine that created the model.
  - `WindowSize`: If the model was estimated by `spa`, `spafdr`, or `etfe`, the size of window (input argument `M`, the resolution parameter) that was used. This is scalar or a vector.
  - `DataName`: Name of the data set from which the model was estimated.
  - `DataLength`: Length of this data set.

Note that you can set or retrieve all properties either with the `set` and `get` commands or by subscripts. Autofill applies to all properties and values, and these are case insensitive:

```
h.ts = 0
loglog(h.fre,squeeze(h.spe(2,2,:)))
```

For a complete list of property values, use `get(m)`. To see possible value assignments, use `set(m)`. See also `idprops idfrd`.

Subreferencing

The different channels of the `idfrd` are retrieved by subreferencing.

```
h(outputs,inputs)
```

`h(2,3)` thus contains the response data from input channel 3 to output channel 2, and, if applicable, the output spectrum data for output channel 2. The channels can also be referred to by their names, as in `h('power',{'voltage',''speed'})`.

```
h('m')
```

contains the information for measured inputs only, that is, just `ResponseData`, while

```
h('n')
```

(`'n'` for `'noise'`) just contains `SpectrumData`.

Horizontal Concatenation

Adding input channels,

```
h = [h1,h2,...,hN]
```

creates an idfrd model h, with ResponseData containing all the input channels in h1,...,hN. The output channels of hk must be the same, as well as the frequency vectors. SpectrumData is ignored.

Vertical Concatenation

Adding output channels,

```
h = [h1;h2;... ;hN]
```

creates an idfrd model h with ResponseData containing all the output channels in h1, h2,...,hN. The input channels of hk must all be the same, as well as the frequency vectors. SpectrumData is also appended for the new outputs. The cross spectrum between output channels is then set to zero.

Converting to iddata

You can convert an idfrd object to a frequency-domain iddata object by

```
Data = iddata(Idfrdmodel)
```

See iddata.

Examples

Compare the results from spectral analysis and an ARMAX model.

```
m = armax(z,[2 2 2 1]);
g = spa(z)
g = spafdr(z,[],{0,10})
bode(g,m)
```

Compute separate idfrd models, one containing the frequency function and the other the noise spectrum.

```
g = idfrd(m('m'))
phi = idfrd(m('n'))
```

See Also

bode, etfe, freqresp, nyquist, spa

Purpose

Construct grey-box linear model using user-defined M-file

Syntax

```
m = idgrey(MfileName,ParameterVector,CDmfile)
m = idgrey(MfileName,ParameterVector,CDmfile,FileArgument,Ts,...
'Property1',Value1,...,'PropertyN',ValueN)
```

Description

The function `idgrey` is used to create arbitrarily parameterized state-space models as `idgrey` objects.

`MfileName` is the name of an M-file that defines how the state-space matrices depend on the parameters to be estimated. The format of this M-file is given by

```
[A,B,C,D,K,X0] = mymfile(pars,Tsm,Auxarg)
```

and is further discussed below.

`ParameterVector` is a column vector of the nominal/initial parameters. Its length must be equal to the number of free parameters in the model (that is, the argument `pars` in the example below).

The argument `CDmfile` describes how the user-written M-file handles continuous and discrete-time models. It takes the following values:

- `CDmfile = 'cd'`: The M-file returns the continuous-time state-space matrices when called with the argument `Tsm = 0`. When called with a value `Tsm > 0`, the M-file returns the discrete-time state-space matrices, obtained by sampling the continuous-time system with sampling interval `Tsm`. The M-file must consequently in this case include the sampling procedure.
- `CDmfile = 'c'`. The M-file always returns the continuous-time state-space matrices, no matter the value of `Tsm`. In this case the toolbox's estimation routines will provide the sampling when you are fitting the model to discrete-time data.
- `CDmfile='d'`. The M-file always returns discrete-time state-space matrices that may or may not depend on `Tsm`.

The argument `FileArgument` corresponds to the auxiliary argument `Auxarg` in the user-written M-file. It can be used to handle several variants of the model structure, without your having to edit the M-file. If it is not used, enter `FileArgument = []`. (Default.)

`Ts` denotes the sampling interval of the model. Its default value is `Ts = 0`, that is, a continuous-time model.

The `idgrey` object is a child of `idmodel`. Therefore any `idmodel` properties can be set as property name/property value pairs in the `idgrey` command. They can also be set by the command `set`, or by subassignment, as in

```
m.InputName = {'speed','voltage'}
m.FileArgument = 0.23
```

There are also two properties, `DisturbanceModel` and `InitialState`, that can be used to affect the parameterizations of *K* and *X0*, thus overriding the outputs from the M-file.

idgrey Properties

- `MfileName`: Name of the user-written M-file.
- `CDmfile`: How this file handles continuous and discrete-time models depending on its second argument, `T`.
  - `CDmfile = 'cd'` means that the M-file returns the continuous-time state-space model matrices when the argument `T = 0`, and the discrete-time model, obtained by sampling with sampling interval `T`, when `T > 0`.
  - `CDmfile= 'c'` means that the M-file always returns continuous-time model matrices, no matter the value of T.
  - `CDmfile = 'd'` means that the M-file always returns discrete-time model matrices that may or may not depend on the value of `T`.
- `FileArgument`: Possible extra input arguments to the user-written M-file.
- `DisturbanceModel`: Affects the parameterization of the K matrix. It can assume the following values:
  - `'Model'`: This is the default. It means that the K matrix obtained from the user-written M-file is used.
  - `'Estimate'`: The K matrix is treated as unknown and all its elements are estimated as free parameters.
  - `'Fixed'`: The K matrix is fixed to a given value.
  - `'None'`: The K matrix is fixed to zero, thus producing an output-error model.

  Note that in the three last cases the output `K` from the user-written M-file is ignored. The estimated/fixed value is stored internally and does not change when the model is sampled, resampled, or converted to continuous time.

Note also that this estimated value is tailored only to the sampling interval of the data.

- `InitialState`: Affects the parameterization of the X0 vector. It can assume the following values:
  - `'Model'`: This is the default. It means that the X0 vector is obtained from the user-written M-file.
  - `'Estimate'`: The X0 matrix is treated as unknown and all its elements are estimated as free parameters.
  - `'Fixed'`: The X0 vector is fixed to a given value.
  - `'Backcast'`: The X0 vector is estimated using a backcast operation analogous to the `idss` case.
  - `'Auto'`: Makes a data-dependent choice among `'Estimate'`, `'Backcast'`, and `'Model'`.
- `A`, `B`, `C`, `D`, `K`, and `X0`: The state-space matrices. For `idgrey` models, only `'K'` and `'X0'` can be set; the others can only be retrieved. The set `'K'` and `'X0'` are relevant only when `DisturbanceModel/InitialState` are `Estimate` or `Fixed`.
- `dA`, `dB`, `dC`, `dD`, `dK`, and `dX0`: The estimated standard deviations of the state-space matrices. These cannot be set, only retrieved.

In addition, any `idgrey` object also has all the properties of `idmodel`. See `Algorithm Properties` and the reference page for `idmodel`.

Note that you can set or retrieve all properties using either the `set` and `get` commands or subscripts. Autofill applies to all properties and values, and they are case insensitive.

```
m.fi = 10;
set(m,'search','gn')
p = roots(m.a)
```

For a complete list of property values, use `get(m)`. To see possible value assignments, use `set(m)`. See also `idprops` and `idgrey`.

M-File Details

The model structure corresponds to the general linear state-space structure

$$\tilde{x}(t) = A(\theta)x(t) + B(\theta)u(t) + K(\theta)e(t)$$

$$x(0) = x_0(\theta)$$

$$y(t) = C(\theta)x(t) + D(\theta)u(t) + e(t)$$

Here $\tilde{x}(t)$ is the time derivative $\dot{x}(t)$ for a continuous-time model and $x(t + Ts)$ for a discrete-time model.

The matrices in this time-discrete model can be parameterized in an arbitrary way by the vector θ. Write the format for the M-file as follows:

```
[A,B,C,D,K,x0] = mymfile(pars,T,Auxarg)
```

Here the vector `pars` contains the parameters θ, and the output arguments `A`, `B`, `C`, `D`, `K`, and `x0` are the matrices in the model description that correspond to this value of the parameters and this value of the sampling interval `T`.

`T` is the sampling interval, and `Auxarg` is any variable of auxiliary quantities with which you want to work. (In that way you can change certain constants and other aspects in the model structure without having to edit the M-file.) Note that the two arguments `T` and `Auxarg` must be included in the function head of the M-file, even if they are not used within the M-file.

"State-Space Models with Coupled Parameters: the idgrey Model" on page 3-51 contains several examples of typical M-files that define model structures.

A comment about `CDmfile`: If a continuous-time model is sought, it is easiest to let the M-file deliver just the continuous-time model, that is, have `CDmfile = 'c'` and rely upon the toolbox's routines for the proper sampling. Similarly, if the underlying parameterization is indeed discrete time, it is natural to deliver the discrete-time model matrices and let `CDmfile = 'd'`. If the underlying parameterization is continuous, but you prefer for some reason to do your own sampling inside the M-file in accordance with the value of `T`, then let your M-file deliver the continuous-time model when called with `T = 0`, that is, the alternative `CMmfile = 'cd'`. This avoids sampling and then transforming back (using `d2c`) to find the continuous-time model.

Examples

Use the M-file `mynoise` given in "Parameterized Disturbance Models" on page 3-53 to obtain a physical parameterization of the Kalman gain.

```
mn = idgrey('mynoise',[0.1,-2,1,3,0.2]','d')
m = pem(z,mn)
```

Purpose

Generate identification input signals

Syntax

```
u = idinput(N)
u = idinput(N,type,band,levels)
[u,freqs] = idinput(N,'sine',band,levels,sinedata)
```

Description

`idinput` generates input signals of different kinds, which are typically used for identification purposes. `u` is returned as a matrix or column vector.

For further use in the toolbox, we recommend that you create an `iddata` object from `u`, indicating sampling time, input names, periodicity, and so on:

```
u = iddata([],u);
```

`N` determines the number of generated input data. If `N` is a scalar, `u` is a column vector with this number of rows.

`N = [N nu]` gives an input with `nu` input channels each of length `N`.

`N = [P nu M]` gives a periodic input with `nu` channels, each of length `M*P` and periodic with period `P`.

Default is `nu = 1` and `M = 1`.

`type` defines the type of input signal to be generated. This argument takes one of the following values:

- `type = 'rgs'`: Gives a random, Gaussian signal.
- `type = 'rbs'`: Gives a random, binary signal. This is the default.
- `type = 'prbs'`: Gives a pseudorandom, binary signal.
- `type = 'sine'`: Gives a signal that is a sum of sinusoids.

The frequency contents of the signal is determined by the argument `band`. For the choices `type = 'rs'`, `'rbs'`, and `'sine'`, this argument is a row vector with two entries

```
band = [wlow, whigh]
```

that determine the lower and upper bound of the passband. The frequencies `wlow` and `whigh` are expressed in fractions of the Nyquist frequency. A white noise character input is thus obtained for `band = [0 1]`, which is also the default value.

For the choice `type = 'prbs'`,

```
band = [0, B]
```

where `B` is such that the signal is constant over intervals of length `1/B` (the clock period). In this case the default is `band = [0 1]`.

The argument `levels` defines the input level. It is a row vector

```
levels = [minu, maxu]
```

such that the signal `u` will always be between the values `minu` and `maxu` for the choices `type = 'rbs'`, `'prbs'`, and `'sine'`. For `type = 'rgs'`, the signal level is such that `minu` is the mean value of the signal, minus one standard deviation, while `maxu` is the mean value plus one standard deviation. Gaussian white noise with zero mean and variance one is thus obtained for `levels = [-1, 1]`, which is also the default value.

Some PRBS Aspects

If more than one period is demanded (that is, `M > 1`), the length of the data sequence and the period of the PRBS signal are adjusted so that an integer number of maximum length PRBS periods is always obtained. If `M = 1`, the period of the PRBS signal is chosen to that it is longer than `P = N`. In the multiinput case, the signals are maximally shifted. This means `P/nu` is an upper bound for the model orders that can be estimated with such a signal.

Some Sine Aspects

In the `'sine'` case, the sinusoids are chosen from the frequency grid

`freq = 2*pi*[1:Grid_Skip:fix(P/2)]/P` intersected with `pi*[band(1) band(2)]`

(for `Grid_Skip`, see below.) For multiinput signals, the different inputs use different frequencies from this grid. An integer number of full periods is always delivered. The selected frequencies are obtained as the second output argument, `freqs`, where row `ku` of `freqs` contains the frequencies of input number `ku`. The resulting signal is affected by a fifth input argument, `sinedata`

```
sinedata = [No_of_Sinusoids, No_of_Trials, Grid_Skip]
```

meaning that `No_of_Sinusoids` is equally spread over the indicated `band`. `No_of_Trials` (different, random, relative phases) are tried until the lowest amplitude signal is found.

```
Default: sinedata = [10,10,1];
```

`Grid_Skip` can be useful for controlling odd and even frequency multiples, for example, to detect nonlinearities of various kinds.

Algorithm

Very simple algorithms are used. The frequency contents are achieved for `'rgs'` by an eighth-order Butterworth, noncausal filter, using `idfilt`. This is quite reliable. The same filter is used for the `'rbs'` case, before making the signal binary. This means that the frequency contents are not guaranteed to be precise in this case.

For the `'sine'` case, the frequencies are selected to be equally spread over the chosen grid, and each sinusoid is given a random phase. A number of trials are made, and the phases that give the smallest signal amplitude are selected. The amplitude is then scaled so as to satisfy the specifications of `levels`.

References

See Söderström and Stoica (1989), Chapter C5.3. For a general discussion of input signals, see Ljung (1999), Section 13.3.

Examples

Create an input consisting of five sinusoids spread over the whole frequency interval. Compare the spectrum of this signal with that of its square. The frequency splitting (the square having spectral support at other frequencies) reveals the nonlinearity involved:

```
u = idinput([100 1 20],'sine',[],[],[5 10 1]);
u = iddata([],u,1,'per',100);
u2 = u.u.^2;
u2 = iddata([],u2,1,'per',100);
ffplot(etfe(u),'r*',etfe(u2),'+')
```

idmdlsim

Purpose

Simulate `idmodel` objects in Simulink

Syntax

```
idmdlsim
```

Description

Typing `idmdlsim` launches the Idmodel block in Simulink. By clicking the block you can specify the `idmodel` to simulate, whether to include initial state values, and whether to add noise to the simulation in accordance with the model's own noise description.

Purpose

Package common model properties

Description

`idmodel` is an object that the user does not deal with directly. It contains all the common properties of the model objects `idarx`, `idgrey`, `idpoly`, `idproc`, and `idss`, which are returned by the different estimation routines.

Basic Use

If you just estimate models from data, the model objects should be transparent. All parametric estimation routines return `idmodel` results.

```
m = arx(Data,[2 2 1])
```

The model `m` contains all relevant information. Just typing `m` will give a brief account of the model. `present(m)` also gives information about the uncertainties of the estimated parameters. `get(m)` gives a complete list of model properties.

Most of the interesting properties can be directly accessed by subreferencing.

```
m.a
m.da
```

See the property list obtained by `get(m)`, as well as the property lists of `idgrey`, `idarx`, `idpoly`, and `idss` in Chapter 4, "Functions — By Category" for more details on this. See also `idprops`.

The characteristics of the model `m` can be directly examined and displayed by commands like `impulse`, `step`, `bode`, `nyquist`, and `pzmap`. The quality of the model is assessed by commands like `compare` and `resid`. If you have the Control System Toolbox, typing `view(m)` gives access to various display functions.

To extract state-space matrices, transfer function polynomials, etc., use the commands `arxdata`, polydata, `tfdata`, `ssdata`, and `zpkdata`.

To compute the frequency response of the model, use the commands `idfrd` and `freqresp`.

Creating and Modifying Model Objects

If you want to define a model to use, for example, for simulating data, you need to use the model creator functions:

- idarx, for multivariable ARX models
- idgrey, for user-defined gray-box state-space models
- idpoly, for single-output polynomial models
- idproc, for simple, continuous-time process models
- idss, for state-space models

If you want to estimate a state-space model with a specific internal parameterization, you need to create an idss model or an idgrey model. See the reference pages for these functions.

Dealing with Input and Output Channels

For multivariable models, you construct submodels containing a subset of inputs and outputs by simple subreferencing. The outputs and input channels can be referenced according to

```
m(outputs,inputs)
```

Use a colon (:) to denote all channels and an empty matrix ([]) to denote no channels. The channels can be referenced by number or by name. For several names, you must use a cell array, such as

```
m3 = m('position',{'power','speed'})
```

or

```
m3 = m(3,[1 4])
```

Thus m3 is the model obtained from m by looking at the transfer functions from input numbers 1 and 4 (with input names 'power' and 'speed') to output number 3 (with name position).

For a single-output model m,

```
m4 = m(inputs)
```

selects the corresponding input channels, and for a single-input model,

```
m5 = m(outputs)
```

selects the indicated output channels.

Subreferencing is quite useful, for example, when a plot of just some channels is desired.

The Noise Channels

The estimated models have two kinds of input channels: the measured inputs u and the noise inputs e. For a general linear model `m`, we have

$$y(t) = G(q)u(t) + H(q)e(t) \qquad \textbf{(4-2)}$$

where `u` is the nu-dimensional vector of measured input channels and `e` is the ny-dimensional vector of noise channels. The covariance matrix of `e` is given by the property `'NoiseVariance'`. Occasionally this matrix Λ is written in factored form,

$$\Lambda = LL^T$$

This means that `e` can be written as

$$e = Lv$$

where v is white noise with identity covariance matrix (independent noise sources with unit variances).

If `m` is a time series (nu `= 0`), G is empty and the model is given by

$$y(t) = H(q)e(t)$$

For the model `m`, the restriction to the transfer function matrix G is obtained by

`m1 = m('measured')` or just `m1 = m('m')`

Then `e` is set to `0` and `H` is removed.

Analogously,

`m2 = m('noise')` or just `m2 = m('n')`

creates a time-series model `m2` from `m` by ignoring the measured input. That is, `m2` describes the signal He.

For a system with measured inputs, `bode`, `step`, and other transformation and display functions deal with the transfer function matrix G. To obtain or graph the properties of the disturbance model H, it is therefore important to make the transformations `m('n')`. For example,

```
bode(m('n'))
```

plots the additive noise spectra according to the model `m`, while

```
bode(m)
```

just plots the frequency responses of G.

To study the noise contributions in more detail, it is useful to convert the noise channels to measured channels, using the command `noisecnv`.

```
m3 = noisecnv(m)
```

This creates a model `m3` with all input channels, both measured `u` and noise sources `e`, treated as measured signals,. That is, `m3` is a model from `u` and `e` to `y`, describing the transfer functions G and H. The information about the variance of the innovations `e` is lost. For example, studying the step response from the noise channels does not take into consideration how large the noise contributions actually are.

To include that information, `e` should first be normalized, $e = Lv$, so that v becomes white noise with an identity covariance matrix.

```
m4 = noisecnv(m,'Norm')
```

This creates a model `m4` with `u` and v treated as measured signals.

$$y(t) = G(q)u(t) + H(q)Lv(t) = \begin{bmatrix} G & HL \end{bmatrix} \begin{bmatrix} u \\ v \end{bmatrix}$$

For example, the step responses from v to y will now reflect the typical size of the disturbance influence because of the scaling by L. In both cases, the previous noise sources that have become regular inputs will automatically get input names that are related to the corresponding output. The unnormalized noise sources `e` have names like `'e@ynam1'` (noise `e` at output channel `ynam1`), while the normalized sources `v` are called `'v@ynam1'`.

Retrieving Transfer Functions

The functions that retrieve transfer function properties, `ssdata`, `tfdata`, and `zpkdata`, will thus work as follows for a model (Equation 4-2) with measured inputs. (`fcn` is `ssdata`, `tfdata`, or `zpkdata`.)

`fcn(m)` returns the properties of G (ny outputs and nu inputs).

`fcn(m('n'))` returns the properties of the transfer function H (ny outputs and ny inputs).

`fcn(noisec nv(m,'Norm'))` returns the properties of the transfer function [*G HL*} (*ny* outputs and *ny*+*nu* inputs). Analogously,

```
m1 = m('n'). fcn(noisecnv(m1,'Norm'))
```

returns the properties of the transfer function *HL* (*ny* outputs and *ny* inputs).

If `m` is a time-series model, `fcn(m)` returns the properties of *H*, while

```
fcn(noisecnv(m,'Norm'))
```

returns the properties of *HL*.

Note that the estimated covariance matrix `NoiseVariance` itself is uncertain. This means that the uncertainty information about *H* is different from that of *HL*.

idmodel Properties

In the list below, `ny` is the number of output channels, and `nu` is the number of input channels:

- `Name`: An optional name for the data set. An arbitrary string.
- `OutputName`, `InputName`: Cell arrays of length `ny`-by-1 and `nu`-by-1 containing the names of the output and input channels. For estimated models, these are inherited from the data. If not specified, they are given default names `{'y1','y2',...}` and `{'u1','u2',...}`.
- `OutputUnit`, `InputUnit`: Cell arrays of length `ny`-by-1 and `nu`-by-1 containing the units of the output and input channels. Inherited from data for estimated models.
- `TimeUnit`: Unit for the sampling interval.
- `Ts`: Sampling interval. A nonnegative scalar. `Ts = 0` denotes a continuous-time model. Note that changing just `Ts` will not recompute the model parameters. Use `c2d` and `d2c` for recomputing the model to other sampling intervals.
- `ParameterVector`: Vector of adjustable parameters in the model structure. Initial/nominal values or estimated values, depending on the status of the model. A column vector.
- `PName`: The names of the parameters. A cell array of the length of the parameter vector. If not specified, it will contain empty strings. See also `setpname`.

- `CovarianceMatrix`: Estimated covariance matrix of the parameter vector. For a nonestimated model this is the empty matrix. For state-space models in the `'Free'` parameterization the covariance matrix is also the empty matrix, since the individual matrix elements are not identifiable then. Instead, in this case, the covariance information is hidden (in the hidden property `'Utility'`) and retrieved by the relevant functions when necessary. Setting `CovarianceMatrix` to `'None'` inhibits calculation of covariance and uncertainty information. This can save substantial time for certain models. See “No Covariance” on page 3-104.
- `NoiseVariance`: Covariance matrix of the noise source `e`. An `ny`-by-`ny` matrix.
- `InputDelay`: Vector of size `nu`-by-1, containing the input delay from each input channel. For a continuous-time model (`Ts = 0`) the delay is measured in `TimeUnit`, while for discrete-time models (`Ts > 0`) the delay is measured as the number of samples. Note the difference between `InputDelay` and `nk` (which is a property of `idarx`, `idss`, and `idpoly`). `'Nk'` is a model structure property that tells the model structure to include such an input delay. In that case, the corresponding state-space matrices and polynomials will explicitly contain `Nk` input delays. The property `InputDelay`, on the other hand, is an indication that in addition to the model as defined, the inputs should be shifted by the given amount. `InputDelay` is used by `sim` and the estimation routines to shift the input data. When computing frequency responses, the `InputDelay` is also respected. Note that `InputDelay` can be both positive and negative.
- `Algorithm`: See the reference page for `Algorithm Properties`.
- `EstimationInfo`: See the reference page for `EstimationInfo`.
- `Notes`: An arbitrary field to store extra information and notes about the object.
- `UserData`: An arbitrary field for any possible use.

Note All properties can be set or retrieved either by these commands or by subscripts. Autofill applies to all properties and values, and is case insensitive.

For a complete list of property values, use `get(m)`. To see possible value assignments, use `set(m)`.

Subreferencing

The outputs and input channels can be referenced according to

```
m(outputs,inputs)
```

Use a colon (`:`) to denote all channels and an empty matrix ([ ]) to denote no channels. The channels can be referenced by number or by name. For several names, you must use a cell array.

```
m2 = m('y3',{'u1','u4'})
```

```
m3 = m(3,[1 4])
```

For a single output model `m`,

```
m4 = m(inputs)
```

selects the corresponding input channels, and for a single input model,

```
m5 = m(outputs)
```

selects the indicated output channels.

The string `'measured'` (or any abbreviation like `'m'`) means the measured input channels.

```
m4 = m(3,'m')
m('m') is the same as m(:,'m')
```

Similarly, the string `'noise'` (or any abbreviation) refers to the noise input channels. See “The Noise Channels” on page 4-99 for more details.

Horizontal Concatenation

Adding input channels,

```
m = [m1,m2,...,mN]
```

creates an `idmodel` object `m`, consisting of all the input channels in `m1,... mN`. The output channels of `mk` must be the same.

Vertical Concatenation

Adding output channels,

```
m = [m1;m2;... ;mN]
```

creates an `idmodel` object `m` consisting of all the output channels in `m1`, `m2`, `..mN`. The input channels of `mk` must all be the same.

Online Help Functions

See `idhelp. idprops idmodel`, `help idmodel/subsref`, `help idmodel/subsasgn`, `help idmodel/horzcat`, and `help idmodel/vertcat`.

See Also

`noisecnv`, `nkshift`, `view`, `size`, `idmdlsim`, `sim`

Purpose

Create structure for input-output models using numerator and denominator polynomials

Syntax

```
m = idpoly(A,B)
m = idpoly(A,B,C,D,F,NoiseVariance,Ts)
m = idpoly(A,B,C,D,F,NoiseVariance,Ts,'Property1',Value1,...
           'PropertyN',ValueN)
m = idpoly(mi)
```

Description

`idpoly` creates a model object containing parameters that describe the general multiinput single-output model structure.

$$A(q)y(t) = \frac{B_1(q)}{F_1(q)}u_1(t - nk_1) + \ldots + \frac{B_{nu}(q)}{F_{nu}(q)}u_{nu}(t - nk_{nu}) + \frac{C(q)}{D(q)}e(t)$$

`A`, `B`, `C`, `D`, and `F` specify the polynomial coefficients.

For single-input systems, these are all row vectors in the standard MATLAB format.

```
A = [1 a1 a2 ... ana]
```

consequently describes

$$A(q) = 1 + a_1 q^{-1} + \ldots + a_{na} q^{-na}$$

`A`, `C`, `D`, and `F` all start with 1, while `B` contains leading zeros to indicate the delays. See “Polynomial Representation of Transfer Functions” on page 3-11.

For multiinput systems, `B` and `F` are matrices with one row for each input.

For time series, `B` and `F` are entered as empty matrices.

```
B = [];  F = [];
```

`NoiseVariance` is the variance of the white noise sequence $e(t)$, while `Ts` is the sampling interval.

Trailing arguments `C`, `D`, `F`, `NoiseVariance`, and `Ts` can be omitted, in which case they are taken as 1. (If `B = []`, then `F` is taken as `[]`.) The property name/property value pairs can start directly after `B`.

`Ts = 0` means that the model is a continuous-time one. Then the interpretation of the arguments is that

```
A = [1 2 3 4]
```

corresponds to the polynomial $s^3 + 2s^2 + 3s + 4$ in the Laplace variable s, and so on. For continuous-time systems, `NoiseVariance` indicates the level of the spectral density of the innovations. A sampled version of the model has the innovations variance `NoiseVariance/Ts`, where `Ts` is the sampling interval. The continuous-time model must have a white noise component in its disturbance description. See "Spectrum Normalization and the Sampling Interval" on page 3-107.

For discrete-time models (`Ts > 0`), note the following: `idpoly` strips any trailing zeros from the polynomials when determining the orders. It also strips leading zeros from the `B` polynomial to determine the delays. Keep this in mind when you use `idpoly` and `polydata` to modify earlier estimates to serve as initial conditions for estimating new structures. See "Initial Parameter Values" on page 3-99.

`idpoly` can also take any single-output `idmodel` or LTI object `mi` as an input argument. If an LTI system has an input group with name `'Noise'`, these inputs are interpreted as white noise with unit variance, and the noise model of the `idpoly` model is computed accordingly.

Properties

- `na`, `nb`, `nc`, `nd`, `nf`, `nk`: The orders and delays of the polynomials. Integers or row vectors of integers.
- `a`, `b`, `c`, `d`, `f`: The polynomials, described by row vectors and matrices as detailed above.
- `da`, `db`, `dc`, `dd`, `df`: The estimated standard deviation of the polynomials. Cannot be set.
- `InitialState`: How to deal with the initial conditions that are required to compute the prediction of the output. Possible values are
  - `'Estimate'`: The necessary initial states are estimated from data as extra parameters.
  - `'Backcast'`: The necessary initial states are estimated by a backcasting (backward filtering) process, described in Knudsen (1994).
  - `'Zero'`: All initial states are taken as zero.
  - `'Auto'`: An automatic choice among the above is made, guided by the data.

In addition, any `idpoly` object also has all the properties of `idmodel`. See `idmodel` properties and `Algorithm Properties`.

Note that you can set or retrieve all properties either with the `set` and `get` commands or by subscripts. Autofill applies to all properties and values, and these are case insensitive.

```
m.a=[1 -1.5 0.7];
set(m,'ini','b')
p = roots(m.a)
```

For a complete list of property values, use `get(m)`. To see possible value assignments, use `set(m)`. See also `idprops idpoly`.

Examples

To create a system of ARMAX, type

```
A = [1 -1.5 0.7];
B = [0 1 0.5];
C = [1 -1 0.2];
m0 = idpoly(A,B,C);
```

This gives a system with one delay (`nk = 1`).

Create the continuous-time model

$$y(t) = \frac{1}{s(s+1)}u_1(t) + \frac{s+3}{s^2+2s+4}u_2(t) + e(t)$$

Sample it with `T = 0.1` and then simulate it without noise.

```
B=[0 1;1 3];
F=[1 1 0;1 2 4]
m = idpoly(1,B,1,1,F,1,0)
md = c2d(m,0.1)
y = sim(md,[u1 u2])
```

Note that the continuous-time model is automatically sampled to the sampling interval of the data, when simulated, so the above is also achieved by

```
u = iddata([],[u1 u2],0.1)
y = sim(m,u)
```

References

Ljung (1999) Section 4.2 for the model structure family.

Knudsen, T., (1994), “A new method for estimating ARMAX models,” In *Proc. 10th IFAC Symposium on System Identification,* pp. 611-617, Copenhagen, Denmark, for the backcast method.

See Also

`sim`, `idss`

Purpose

Create simple, continuous-time process models

Syntax

```
m = idproc(Type)
m = idproc(Type,'Property1',Value1,...,'PropertyN',ValueN)
m = pem(Data,Type) % to directly estimate an idproc model
```

Description

The function `idproc` is used to create typical simple, continuous-time process models as `idproc` objects. The model has one output, but can have several inputs.

The character of the model is defined by the argument `Type`. This is an acronym made up of the following symbols:

- P: All `'Type'` acronyms start with this letter.
- 0, 1, 2, or 3: This integer denotes the number of time constants (poles) to be modeled. Possible integrations (poles in the origin) are not included in this number.
- I: The letter I is included to mark that an integration is enforced (self-regulation process).
- D: The letter D is used to mark that the model contains a time delay (dead time).
- Z: The letter Z is used to mark an extra numerator term: a zero.
- U: The letter U is included to mark that underdamped modes (complex-valued poles) are permitted. If U is not included, all poles are restricted to be real.

This means, for example, that `Type = 'P1D'` corresponds to the model with transfer function

$$G(s) = \frac{K_p}{1 + sT_{p1}} e^{-T_d s}$$

while `Type = 'P0I'` is

$$G(s) = \frac{K_p}{s}$$

and `Type = 'P3UZ'` is

$$G(s)= K_p \frac{1+T_z s}{(1+2\zeta T_w s+(T_w s)^2)(1+T_{p3}s)}$$

For multiinput systems, `Type` is a cell array where each cell describes the character of the model from the corresponding input, like `Type = {'P1D','POI'}` for the two-input model

$$Y(s) = \frac{K_p(1)}{1+sT_{p1}(1)}e^{-T_d s}U_1(s)+\frac{K_p(2)}{s}U_2(s) \qquad \textbf{(4-3)}$$

The parameters of the model are

- `Kp`: The static gain
- `Tp1`, `Tp2`, `Tp3`: The real-time constants (corresponding to poles in `1/Tp1`, etc.)
- `Tw` and `Zeta`: The "resonance time constant" and the damping factor corresponding to a denominator factor `(1+2 Zeta Tw s + (Tw s)^2)`. If underdamped modes are allowed, `Tw` and `Zeta` replace `Tp1` and `Tp2`. A third real pole, `Tp3`, could still be included.
- `Td`: The time delay
- `Tz`: The numerator zero

These properties contain fields that give the values of the parameters, upper and lower bounds, and information whether they are locked to zero, have a fixed value, or are to be estimated. For multiinput models, the number of entries in these fields equals the number of inputs. This is described in more detail below.

The `idproc` object is a child of `idmodel`. Therefore any `idmodel` properties can be set as property name/property value pairs in the `idproc` command. They can also be set by the command `set`, or by subassignment, as in

```
m.InputName = {'speed','voltage'}
m.kp = 12
```

In the multiinput case, models for specific inputs can be obtained by regular subreferencing.

```
m(ku)
```

There are also two properties, `DisturbanceModel` and `InitialState`, that can be used to expand the model. See below.

idproc Properties

- `Type`: A string or a cell array of strings with as many elements as there are inputs. The string is an acronym made up of the characters `P`, `Z`, `I`, `U`, `D` and an integer between 0 and 3. The string must start with `P`, followed by the integer, while possible other characters can follow in any order. The integer is the number of poles (not counting a possible integration), `Z` means the inclusion of a numerator zero, `D` means inclusion of a time delay, while `U` marks that the modes can be underdamped (a pair of complex conjugated poles). `I` means that an integration in the model is enforced.
- `Kp`, `Tp1`, `Tp2`, `Tp3`, `Tw`, `Zeta`, `Tz`, `Td`: These are the parameters as explained above. Each of these is a structure with the following fields:
  - `value`: Numerical value of the parameter.
  - `max`: Maximum allowed value of the parameter when it is estimated.
  - `min`: Minimum allowed value of the parameter when it is estimated. For multiinput models, these are row vectors.
  - `status`: Assumes one of `'Estimate'`, `'Fixed'`, or `'Zero'`.

    `'Zero'` means that the parameter is locked to zero and not included in the model. Assigning, for example, `Type = 'P1'` means that the status of `Tp2`, `Tp3`, `Tw`, and `Zeta` will be `'Zero'`.

    The value `'Fixed'` means that the parameter is fixed to its value, and will not be estimated.

    The value `'Estimate'` means that the parameter value should be estimated.

    For multiinput modes, `status` is a cell array with one element for each input, while `value`, `max`, and `min` are row vectors.
- `DisturbanceModel`: Allows an additive disturbance model as in

$$y(t) = G(s)u(t) + \frac{C(s)}{D(s)}e(t) \qquad \textbf{(4-4)}$$

  where $G(s)$ is a process model and $e(t)$ is white noise, and C/D is a first- or second-order transfer function.

  `DisturbanceModel` can assume the following values:
  - `'None'`: This is the default. No disturbance model is included (that is, $C=D=1$).

- `'arma1'`: The disturbance model is a first-order ARMA model (that is, *C* and *D* are first-order polynomials).
- `'arma2'` or `'Estimate'`: The disturbance model is a second-order ARMA model (that is, *C* and *D* are second-order polynomials).

When a disturbance model has been estimated, the property `DisturbanceModel` is returned as a cell array, with the first entry being the status as just defined, and the second entry being the actual model, delivered as a continuous-time `idpoly` object.

- `InitialState`: Affects the parameterization of the initial values of the states of the model. It assumes the same values as for other models:
  - `'Zero'`: The initial states are fixed to zero.
  - `'Estimate'`: The initial states are treated as parameters to be estimated.
  - `'Backcast'`: The initial state vector is adjusted, during the parameter estimation step, to a suitable value, but it is not stored.
  - `'Auto'`: Makes a data-dependent choice among the values above.
- `InputLevel`: The offset level of the input signal(s). This is of particular importance for those input channels that contain an integration. `InputLevel` will then define the level from which the integration takes place, and that cannot be handled by estimating initial states. `InputLevel` has the same structure as the model parameters `Kp`, etc., and thus contains the following fields:
  - `value`: Numerical value of the parameter. For multiinput models, this is a row vector.
  - `max`: Maximum allowed value of the parameter when it is estimated.
  - `min`: Minimum allowed value of the parameter when it is estimated. For multiinput models, these are row vectors.
  - `status`: Assumes one of `'Estimate'`, `'Fixed'`, or `'Zero'` with the same interpretations.

In addition, any `idproc` object also has all the properties of `idmodel`. See `Algorithm Properties`, `EstimationInfo`, and `idmodel`.

Note that all properties can be set or retrieved using either the `set` and `get` commands or subscripts. Autofill applies to all properties and values, and these are case insensitive. Also `'u'` and `'y'` are short for `'Input'` and `'Output'`, respectively. You can also set all properties at estimation time as property name/property value pairs in the call to `pem`. An extended syntax allows direct

setting of the fields of the parameter values, so that assigning a numerical value is automatically attributed to the value field, while a string is attributed to the status field.

```
m.kp = 10
m.tp1 = 'estimate'
m = pem(Data,'P1D','kp',10) % initializing the parameter Kp in 10
m = pem(Data,'P1D','kp',10,'kp','fix') % fixing the parameter Kp
to the value 10
m.= pem(Data,'P2U','kp',{'max',4},'kp',{'min',3}) % constraining
Kp to lie between 3 and 4.
m = pem(Data,{'P2I','P1D',},'ulevel',{'est','zer'}) % two inputs,
estimate the offset level

% of the first one
m = pem(Data,'P2U','dist','est') % estimate a noise model
m = pem(Data,'P2U','dist',{'fix',noimod}) % use a fixed
noisemodel, given by the continuous-time idpoly model noimod
m.kp.min(2) = 12 % (minimum Kp for the second input)
m.kp.status{2} = 'fix' % fixing the gain for the second input.
```

For a complete list of property values, use `get(m)`. To see possible value assignments, use `set(m)`. See also `idprops` and `idproc`.

Examples

```
m = pem(Data,'P2D','dist','arma1')
```

Purpose

Create structure for linear state-space models with known and unknown parameters

Syntax

```
m = idss(A,B,C,D)
m = idss(A,B,C,D,K,x0,Ts,'Property1',Value1,...,'PropertyN',ValueN)
mss = idss(m1)
```

Description

The function `idss` is used to construct state-space model structures with various parameterizations. It is a complement to `idgrey` and deals with parameterizations that do not require the user to write a special M-file. Instead it covers parameterizations that are either `'Free'`, that is, all parameters in the A, B, and C matrices can be adjusted freely, or `'Canonical'`, meaning that the matrices are parameterized as canonical forms. The parameterization can also be `'Structured'`, which means that certain elements in the state-space matrices are free to be adjusted, while others are fixed. This is explained below.

`Ts` is the sampling interval. `Ts = 0` means a continuous-time model. The default is `Ts = 1`.

The `idss` object `m` describes state-space models in innovations form of the following kind:

$$\tilde{x}(t) = A(\theta)x(t) + B(\theta)u(t) + K(\theta)e(t)$$

$$x(0) = x_0(\theta)$$

$$y(t) = C(\theta)x(t) + D(\theta)u(t) + e(t)$$

Here $\tilde{x}(t)$ is the time derivative $\dot{x}(t)$ for a continuous-time model and $x(t+Ts)$ for a discrete-time model.

The model `m` will contain information both about the nominal/initial values of the A, B, C, D, K, and X0 matrices and about how these matrices are parameterized by the parameter vector θ (to be estimated).

The nominal model is defined by `idss(A,B,C,D,K,X0)`. If `K` and `X0` are omitted, they are taken as zero matrices of appropriate dimensions.

Defining an `idss` object from a given model,

```
mss = idss(m1)
```

constructs an `idss` model from any `idmodel` or LTI system `m1`.

If `m1` is an LTI system (`ss`, `tf`, or `zpk`) that has no `InputGroup` called `'Noise'`, the corresponding state-space matrices A, B, C, D are used to define the `idss` object. The Kalman gain K is then set to zero.

If the LTI system has an `InputGroup` called `'Noise'`, these inputs are interpreted as white noise with a covariance matrix equal to the identity matrix. The corresponding Kalman gain and noise variance are then computed and entered into the `idss` model together with A, B, C, and D.

Parameterizations

There are several different ways to define the parameterization of the state-space matrices. The parameterization determines which parameters can be adjusted to data by the parameter estimation routine `pem`.

- Free black-box parameterizations: This is the default situation and corresponds to letting all parameters in A, B, and C be freely adjustable. You do this by setting the property `'SSParameterization' = 'Free'`. The parameterizations of D, K, and *X0* are then determined by the following properties:
  - `'nk'`: A row vector of the same length as the number of inputs. The `ku`th element is the delay from input channel number `ku`. Thus `nk = [0,...,0]` means that there is no delay from any of the inputs, and that consequently all elements of the D matrix should be estimated. `nk =[1,...,1]` means that there is a delay of 1 from each input, so that the D matrix is fixed to be zero.
  - `'DisturbanceModel'`: This property affects the parameterization of K and can assume the following values:

    `'Estimate'`: All elements of the K matrix are to be estimated.

    `'None'`: All elements of K are fixed to zero.

    `'Fixed'`: All elements of K are fixed to their nominal/initial values.

- 'InitialState': Affects the parameterization of *X0* and can assume the following values:

  'Auto': An automatic choice of the following is made, depending on data (default).

  'Estimate': All elements of *X0* are to be estimated.

  'Zero': All elements of *X0* are fixed to zero.

  'Fixed': All elements of *X0* are fixed to their nominal/initial values.

  'Backcast': The vector *X0* is adjusted, during the parameter estimation step, to a suitable value, but it is not stored as an estimation result.

- Canonical black-box parameterizations: You do this by setting the property 'SSParameterization' = 'Canonical'. The matrices *A*, *B*, and *C* are then parameterized as an observer canonical form, which means that ny (number of output channels) rows of *A* are fully parameterized while the others contain 0's and 1's in a certain pattern. The *C* matrix is built up of 0's and 1's while the *B* matrix is fully parameterized. See Equation (A.16) in Ljung (1999) for details. The exact form of the parameterization is affected by the property 'CanonicalIndices'. The default value 'Auto' is a good choice. The parameterization of the *D*, *K*, and *X0* matrices in this case is determined by the properties 'nk', 'DisturbanceModel', and 'InitialState'.
- Arbitrarily structured parameterizations: The general case, where arbitrary elements of the state-space matrices are fixed and others can be freely adjusted, corresponds to the case 'SSParameterization' = 'Structured'. The parameterization is determined by the idss properties As, Bs, Cs, Ds, Ks, and X0s. These are the *structure matrices* that are "shadows" of the state-space matrices, so that an element in these matrices that is equal to NaN indicates a freely adjustable parameter, while a numerical value in these matrices indicates that the corresponding system matrix element is fixed (nonadjustable) to this value.

idss Properties

- SSParameterization has the following possible values:
  - 'Free': Means that all parameters in *A*, *B*, and *C* are freely adjustable, and the parameterizations of *D*, *K*, and *X0* depend on the properties 'nk', 'DisturbanceModel', and 'InitialState'.
  - 'Canonical': Means that *A* and *C* are parameterized as an observer canonical form. The details of this parameterization depend on the property 'CanonicalIndices'. The *B* matrix is always fully

parameterized, and the parameterizations of *D*, *K*, and *X0* depend on the properties `'nk'`, `'DisturbanceModel'`, and `'InitialState'`.

- `'Structured'`: Means that the parameterization is determined by the properties (the structure matrices) `'As'`, `'Bs'`, `'Cs'`, `'Ds'`, `'Ks'`, and `'X0s'`. A `NaN` in any position in these matrices denotes a freely adjustable parameter, and a numeric value denotes a fixed and nonadjustable parameter.

- `nk`: A row vector with as many entries as the number of input channels. The entry number `k` denotes the time delay from input number `k` to `y(t)`. This property is relevant only for `'Free'` and `'Canonical'` parameterizations. If any delay is larger than 1, the structure of the *A*, *B*, and *C* matrices will accommodate this delay, at the price of a higher-order model.
- `DisturbanceModel` has the following possible values:
  - `'Estimate'`: Means that the *K* matrix is fully parameterized.
  - `'None'`: Means that the *K* matrix is fixed to zero. This gives a so-called output-error model, since the model output depends on past inputs only.
  - `'Fixed'`: Means that the *K* matrix is fixed to the current nominal values.
- `InitialState` has the following possible values:
  - `'Estimate'`: Means that *X0* is fully parameterized.
  - `'Zero'`: Means that *X0* is fixed to zero.
  - `'Fixed'`: Means that *X0* is fixed to the current nominal value.
  - `'Backcast'`: The value of *X0* is estimated by the identification routines as the best fit to data, but it is not stored.
  - `'Auto'`: Gives an automatic and data-dependent choice among `'Estimate'`, `'Zero'`, and `'Backcast'`.
- `A`, `B`, `C`, `D`, `K`, and `X0`: The state-space matrices that can be set and retrieved at any time. These contain both fixed values and estimated/nominal values.
- `dA`, `dB`, `dC`, `dD`, `dK`, and `dX0`: The estimated standard deviations of the state-space matrices. These cannot be set, only retrieved. Note that these are not defined for an `idss` model with `'Free' SSParameterization`. You can then convert the parameterization to `'Canonical'` and study the uncertainties of the matrix elements in that form.
- `As`, `Bs`, `Cs`, `Ds`, `Ks`, and `X0s`: These are the structure matrices that have the same sizes as `A`, `B`, `C`, etc., and show the freely adjustable parameters as `NaN`s in the corresponding position. These properties are used to define the model

structure for `'SSParameterization' = 'Structured'`. They are always defined, however, and can be studied also for the other parameterizations.

- `CanonicalIndices`: Determines the details of the canonical parameterization. It is a row vector of integers with as many entries as there are outputs. They sum up to the system order. This is the so-called pseudocanonical multiindex with an exact definition, for example, on page 132 in Ljung (1999). A good default choice is `'Auto'`. This property is relevant only for the canonical parameterization case. Note however, that for `'Free'` parameterizations, the estimation algorithms also store a canonically parameterized model to handle the model uncertainty.

In addition to these properties, `idss` objects also have all the properties of the `idmodel` object. See `idmodel` properties, `Algorithm Properties`, and `EstimationInfo`.

Note that all properties can be set and retrieved either by the `set` and `get` commands or by subscripts. Autofill applies to all properties and values, and these are case insensitive.

```
m.ss='can'
set(m,'ini','z')
p = eig(m.a)
```

For a complete list of property values, use `get(m)`. To see possible value assignments, use `set(m)`. See also `idprops idss`.

Examples

Define a continuous-time model structure corresponding to

$$\dot{x} = \begin{bmatrix} \theta_1 & 0 \\ 0 & \theta_2 \end{bmatrix} x + \begin{bmatrix} \theta_3 \\ \theta_4 \end{bmatrix} u$$

$$y = \begin{bmatrix} 1 & 1 \end{bmatrix} x + e$$

with initial values

$$\theta = \begin{bmatrix} -0.2 \\ -0.3 \\ 2 \\ 4 \end{bmatrix}$$

and estimate the free parameters.

```
A = [-0.2, 0; 0, -0.3]; B = [2;4]; C=[1, 1]; D = 0
m0 = idss(A,B,C,D);
m0.As = [NaN,0;0,NaN];
m0.Bs = [NaN;NaN];
m0.Cs = [1,1];
m0.Ts = 0;
m = pem(z,m0);
```

Estimate a model in free parameterization. Convert it to continuous time, then convert it to canonical form and continue to fit this model to data.

```
m1 = n4sid(data,3);
m1 = d2c(m1);
m1.ss ='can';
m = pem(data,m1);
```

All of this can be done at once by

```
m = pem(data,3,'ss','can','ts',0)
```

See Also

`n4sid`, `pem`, `setstruc`

impulse

Purpose

Plot impulse response with confidence regions

Syntax

```
impulse(m)
impulse(data)
impulse(m,'sd',sd,Time)
impulse(m,'sd',sd,Time,'fill')
impulse(data,'sd',sd,'pw',na,Time)
impulse(m1,m2,...,dat1, ...,mN,Time,'sd',sd)
impulse(m1,'PlotStyle1',m2,'PlotStyle2',...,dat1,'PlotStylek',...,
   mN,'PlotStyleN',Time,'sd',sd)
[y,t,ysd] = impulse(m)
mod = impulse(data)
```

Description

`impulse` can be applied both to `idmodels` and to `iddata` sets, as well as to any mixture.

For a discrete-time `idmodel` `m`, the impulse response `y` and, when required, its estimated standard deviation `ysd`, are computed using `sim`. When called with output arguments, `y`, `ysd`, and the time vector `t` are returned. When `impulse` is called without output arguments, a plot of the impulse response is shown. If `sd` is given a value larger than zero, a confidence region around zero is drawn. It corresponds to the confidence of `sd` standard deviations. In the plots, the impulse is inversely scaled with the sampling interval so that it has the same energy regardless of the sampling interval.

Adding an argument `'fill'` among the input arguments gives an uncertainty region marked by a filled area rather than by dash-dotted lines.

Setting the Time Interval

You can specify the start time `T1` and the end time `T2` using `Time= [T1 T2]`. If `T1` is not given, it is set to `-T2/4`. The negative time lags (the impulse is always assumed to occur at time 0) show possible feedback effects in the data when the impulse is estimated directly from data. If `Time` is not specified, a default value is used.

Estimating the Impulse Response from data

For an `iddata` set data, `impulse(data)` estimates a high-order, noncausal FIR model after first having prefiltered the data so that the input is “as white as possible.” The impulse response of this FIR model and, when asked for, its

confidence region, are then plotted. Note that it is not always possible to deliver the demanded time interval when the response is estimated. A warning is then issued. When called with an output argument, `impulse`, in the `iddata` case, returns this FIR model, stored as an `idarx` model. The order of the prewhitening filter can be specified by the property name/property value pair `'pw'/na`. The default value is `na = 10`.

Several Models/Data Sets

Any number and any mixture of models and data sets can be used as input arguments. The responses are plotted with each input/output channel (as defined by the model and data set `InputName` and `OutputName` properties) as a separate plot. Colors, line styles, and marks can be defined by `PlotStyle` values. These are the same as for the regular `plot` command, as in

```
impulse(m1,'b-*',m2,'y--',m3,'g')
```

Noise Channels

The noise input channels in `m` are treated as follows: Consider a model `m` with both measured input channels u (nu channels) and noise channels e (ny channels) with covariance matrix Λ

$$y = Gu + He$$
$$\mathrm{cov}(e) = \Lambda = LL'$$

where L is a lower triangular matrix. Note that `m.NoiseVariance` = Λ. The model can also be described with unit variance, normalized noise source v:

$$y = Gu + HLv$$
$$\mathrm{cov}(v) = I$$

- `impulse(m)` plots the impulse response of the transfer function G.
- `impulse(m('n'))` plots the impulse response of the transfer function H (ny inputs and ny outputs).The input channels have names `e@yname`, where `yname` is the name of the corresponding output.
- If `m` is a time series, that is nu = `0`, `impulse(m)` plots the impulse response of the transfer function H.
- `impulse(noisecnv(m))` plots the impulse response of the transfer function $[G\ H]$ (nu+ny inputs and ny outputs). The noise input channels have names `e@yname`, where `yname` is the name of the corresponding output.

- `impulse(noisecnv(m,'norm'))` plots the impulse response of the transfer function [*G HL*] (*nu+ny* inputs and *ny* outputs). The noise input channels have names `v@yname`, where `yname` is the name of the corresponding output.

Arguments

If `impulse` is called with a single `idmodel` `m`, the output argument `y` is a 3-D array of dimension `Nt`-by-`ny`-by-`nu`. Here `Nt` is the length of the time vector `t`, `ny` is the number of output channels, and `nu` is the number of input channels. Thus `y(:,ky,ku)` is the response in output `ky` to an impulse in the `ku`th input channel.

`ysd` has the same dimensions as `y` and contains the standard deviations of `y`.

If `impulse` is called with an output argument and a single data set in the input arguments, the output is returned as an `idarx` model `mod` containing the high-order FIR model and its uncertainty. By calling `impulse` with `mod`, the responses can be displayed and returned without your having to redo the estimation.

Examples

`impulse(data,'sd',3)` estimates and plots the impulse response. To take a closer look at subsystems, do the following:

```
mod = impulse(data)
impulse(mod(2,3),'sd',3)
```

See Also

`cra`, `step`

Purpose

Set or randomize initial parameter values

Syntax

```
m = init(m0)
m = init(m0,R,pars,sp)
```

Description

This function randomizes initial parameter estimates for model structures `m0` for any idmodel type. `m` is the same model structure as `m0`, but with a different nominal parameter vector. This vector is used as the initial estimate by `pem`.

The parameters are randomized around `pars` with variances given by the row vector `R`. Parameter number k is randomized as `pars(k) + e*sqrt(R(k))`, where `e` is a normal random variable with zero mean and a variance of 1. The default value of `R` is all ones, and the default value of `pars` is the nominal parameter vector in `m0`.

Only models that give stable predictors are accepted. If `sp = 'b'`, only models that are both stable and have stable predictors are accepted.

`sp = 's'` requires stability only of the model, and `sp = 'p'` requires stability only of the predictor. `sp = 'p'` is the default.

Sufficiently free parameterizations can be stabilized by direct means without any random search. To just stabilize such an initial model, set `R = 0`. With `R > 0`, randomization is also done.

For model structures where a random search is necessary to find a stable model/predictor, a maximum of 100 trials is made by `init`. It can be difficult to find a stable predictor for high-order systems by trial and error.

See Also

`idss`, `n4sid`, `pem`

isreal

Purpose

Determine whether model or data set contains real parameters or data

Syntax

```
isreal(Data)
isreal(Model)
```

Description

`Data` is an `iddata` set and `Model` is any `idmodel`. The `isreal` function returns `1` if all parameters of the model are real and if all signals of the data set are real.

See Also

`realdata`

Purpose

Estimate AR model using instrumental variable methods

Syntax

```
m = ivar(y,na)
m = ivar(y,na,nc,maxsize)
```

Description

The parameters of an AR model structure

$$A(q)y(t) = v(t)$$

are estimated using the instrumental variable method. `y` is the signal to be modeled, entered as an `iddata` object (outputs only). `na` is the order of the A polynomial (the number of A parameters). The resulting estimate is returned as an `idpoly` model `m`. The routine is for scalar time-domain signals only.

In the above model, $v(t)$ is an arbitrary process, assumed to be a moving average process of order `nc`, possibly time varying. (Default is `nc = na`.) Instruments are chosen as appropriately filtered outputs, delayed `nc` steps.

The optional argument `maxsize` is explained under `Algorithm Properties`.

Examples

Compare spectra for sinusoids in noise, estimated by the IV method and by the forward-backward least squares method.

```
y = iddata(sin([1:500]'*1.2) + sin([1:500]'*1.5) +
0.2*randn(500,1),[]);
miv = ivar(y,4);
mls = ar(y,4);
bode(miv,mls)
```

References

Stoica, P., et al., *Optimal Instrumental variable estimates of the AR-parameters of an ARMA process*, IEEE Trans. Autom. Control, Vol. AC-30, 1985, pp. 1066-1074.

See Also

`ar`, `etfe`, `spa`

ivstruc

Purpose

Compute loss functions for sets of output-error model structures

Syntax

```
v = ivstruc(ze,zv,NN)
v = ivstruc(ze,zv,NN,p,maxsize)
```

Description

`NN` is a matrix that defines a number of different structures of the ARX type. Each row of `NN` is of the form

```
nn = [na nb nk]
```

with the same interpretation as described for `arx`. See `struc` for easy generation of typical `NN` matrices for single-input systems.

`ze` and `zv` are `iddata` objects containing output-input data. Only time-domain data is supported. Models for each model structure defined in `NN` are estimated using the instrumental variable (IV) method on data set `ze`. The estimated models are simulated using the inputs from data set `zv`. The normalized quadratic fit between the simulated output and the measured output in `zv` is formed and returned in `v`. The rows below the first row in `v` are the transpose of `NN`, and the last row contains the logarithms of the condition numbers of the IV matrix

$$\sum \zeta(t)\varphi^T(t)$$

A large condition number indicates that the structure is of unnecessarily high order (see page 498 in Ljung (1999)).

The information in `v` is best analyzed using `selstruc`.

If `p` is equal to zero, the computation of condition numbers is suppressed. For the use of `maxsize`, see `Algorithm Properties`.

The routine is for single-output systems only.

Note The IV method used does not guarantee that the models obtained are stable. The output-error fit calculated in `v` can then be misleading.

Examples

Compare the effect of different orders and delays, using the same data set for both the estimation and validation.

```
v = ivstruc(z,z,struc(1:3,1:2,2:4));
nn = selstruc(v)
m = iv4(z,nn);
```

Algorithm

A maximum-order ARX model is computed using the least squares method. Instruments are generated by filtering the input(s) through this model. The models are subsequently obtained by operating on submatrices in the corresponding large IV matrix.

See Also

`arxstruc`, `iv4`, `n4sid`, `selstruc`, `struc`

ivx

Purpose

Estimate parameters of ARX model using the instrumental variable (IV) method with arbitrary instruments

Syntax

```
m = ivx(data,orders,x)
m = ivx(data,orders,x,maxsize)
```

Description

`ivx` is a routine analogous to the `iv4` routine, except that you can use arbitrary instruments. These are contained in the matrix `x`. Make this the same size as the output, `data.y`. In particular, if data contains several experiments, `x` must be a cell array with one matrix/vector for each experiment. The instruments used are then analogous to the regression vector itself, except that `y` is replaced by `x`.

Note that `ivx` does not return any estimated covariance matrix for `m`, since that requires additional information. `m` is returned as an `idpoly` object for single-output systems and as an `idarx` object for multioutput systems.

Use `iv4` as the basic IV routine for ARX model structures. The main interest in `ivx` lies in its use for nonstandard situations, for example, when there is feedback present in the data, or when other instruments need to be tried out. Note that there is also an IV version that automatically generates instruments from certain filters you define (type `help iv`).

References

Ljung (1999), page 222.

See Also

`iv4`, `ivar`

Purpose

Estimate ARX model using four-stage instrumental variable method

Syntax

```
m = iv4(data,orders)
m = iv4(data,'na',na,'nb',nb,'nk',nk)
m= iv4(data,orders,'Property1',Value1,...,'PropertyN',ValueN)
```

Description

This function is an alternative to `arx` and the use of the arguments is entirely analogous to the `arx` function. The main difference is that the procedure is not sensitive to the color of the noise term $e(t)$ in the model equation.

For an interpretation of the loss function (innovations covariance matrix), see "Interpretation of the Loss Function" on page 3-109.

Examples

Here is an example of a two-input, one-output system with different delays on the inputs u_1 and u_2.

```
z = iddata(y, [u1 u2]);
nb = [2 2];
nk = [0 2];
m= iv4(z,[2 nb nk]);
```

Algorithm

The first stage uses the `arx` function. The resulting model generates the instruments for a second-stage IV estimate. The residuals obtained from this model are modeled as a high-order AR model. At the fourth stage, the input-output data is filtered through this AR model and then subjected to the IV function with the same instrument filters as in the second stage.

For the multioutput case, optimal instruments are obtained only if the noise sources at the different outputs have the same color. The estimates obtained with the routine are reasonably accurate, however, even in other cases.

References

Ljung (1999), equations (15.21) through (15.26).

See Also

`arx`, `oe`

LTI Commands

Purpose

Allow direct calls to LTI commands from `idmodel` objects (requires Control System Toolbox)

Syntax

```
append, augstate, balreal, canon, d2d, feedback, inv, minreal,
  modred, norm, parallel, series, ss2ss
```

Description

If you have the Control System Toolbox, most of the relevant LTI commands, listed above, can be directly applied to any `idmodel` (`idarx`, `idgrey`, `idpoly`, `idss`). You can also use the overloaded operations +, -, and *. The same operations are performed and the result is delivered as an `idmodel`. The original covariance information is lost most of the time, however.

Examples

You have two more or less identical processes connected in series. Estimate a model for one of them, and use that to form an initial estimate for a model of the connected process.

```
m = pem(data) % data concerns one of the processes
m2 = pem(data2,m*m) % data2 is from the whole connected process
```

Purpose

Merge data sets into one iddata object

Syntax

```
dat = merge(dat1,dat2,....,datN)
```

Description

dat collects the data sets in dat1,.. datN into one iddata object, with several *experiments*. The number of experiments in dat will be the sum of the number of experiments in datk. For the merging to be allowed, a number of conditions must be satisfied:

- All of datk must have the same number of input channels, and the InputNames must be the same.
- All of datk must have the same number of output channels, and the OutputNames must be the same. If some input or output channel is lacking in one experiment, it can be replaced by a vector of NaNs to conform with these rules.
- If the ExperimentNames of datk have been specified as something other than the default 'Exp1', 'Exp2', etc., they must all be unique. If default names overlap, they are modified so that dat will have a list of unique ExperimentNames.

The sampling intervals, the number of observations, and the input properties (Period, InterSample) might be different in the different experiments.

You can retrieve the individual experiments by using the command getexp. You can also retrieve them by subreferencing with a fourth index.

```
dat1 = dat(:,:,:,ExperimentNumber) or

dat1 = dat(:,:,:,ExperimentName)
```

Storing multiple experiments as one iddata object can be very useful for handling experimental data that has been collected on different occasions, or when a data set has been split up to remove "bad" portions of the data. All the toolbox's routines accept multiple-experiment data.

Examples

Bad portions of data have been detected around sample 500 and between samples 720 to 730. Cut out these bad portions and form a multiple-experiment data set that can be used to estimate models without the bad data destroying the estimate.

```
dat = merge(dat(1:498),dat(502:719),dat(719:1000))
```

```
m = pem(dat)
```

Use the first two parts to estimate the model and the third one for validation.

```
m = pem(getexp(dat,[1,2]));
compare(getexp(dat,3),m)
```

See also `iddemo` #9.

See Also

`iddata`, `getexp`

Purpose

Merge estimated models

Syntax

```
m = merge(m1,m2,....,mN)
[m,tv] = merge(m1,m2)
```

Description

The models `m1,m2,...,mN` must all be of the same structure, just differing in parameter values and covariance matrices. Then `m` is the merged model, where the parameter vector is a statistically weighted mean (using the covariance matrices to determine the weights) of the parameters of `mk`.

When two models are merged,

```
[m, tv] = merge(m1,m2)
```

returns a test variable `tv`. It is χ^2 distributed with `n` degrees of freedom, if the parameters of `m1` and `m2` have the same means. Here `n` is the length of the parameter vector. A large value of `tv` thus indicates that it might be questionable to merge the models.

Merging models is an alternative to merging data sets and estimating a model for the merged data. Consequently,

```
m1 = arx(z1,[2 3 4]);
m2 = arx(z2,[2 3 4]);
ma = merge(m1,m2);
```

and

```
mb = arx(merge(z1,z2),[2 3 4]);
```

lead to models `ma` and `mb` that are related and should be close. The difference is that merging the data sets assumes that the signal-to-noise ratios are about the same in the two experiments. Merging the models allows one model to be much more uncertain, for example, due to more disturbances in that experiment. If the conditions are about the same, we recommend that you merge data rather than models, since this is more efficient and typically involves better conditioned calculations.

midprefs

Purpose

Set directory for storing idprefs.mat containing GUI startup information

Syntax

```
midprefs
midprefs(path)
```

Description

The graphical user interface `ident` allows a large number of variables for customized choices. These include the window layout, the default choices of plot options, and names and directories of the four most recent sessions with `ident`. This information is stored in the file `idprefs.mat`, which should be placed on the user's `MATLABPATH`. The default, automatic location for this file is in the same directory as the user's `startup.m` file.

`midprefs` is used to select or change the directory where you store `idprefs.mat`. Either type `midprefs` and follow the instructions, or give the directory name as the argument. Include all directory delimiters, as in the PC case

```
midprefs('c:\matlab\toolbox\local\')
```

or in the UNIX case

```
midprefs('/home/ljung/matlab/')
```

See Also

`ident`

Purpose

Reconstruct missing input and output data

Syntax

```
Datae = misdata(Data)
Datae = misdata(Data,Model)
Datae = misdata(Data,Maxiter,Tol)
```

Description

`Data` is time-domain input-output data in the `iddata` object format. Missing data samples (both in inputs and in outputs) are entered as `NaN`s.

`Datae` is an `iddata` object where the missing data has been replaced by reasonable estimates.

`Model` is any `idmodel` (`idarx`, `idgrey`, `idpoly`, `idss`) used for the reconstruction of missing data.

If no suitable model is known, it is estimated in an iterative fashion using default order state-space models.

`Maxiter` is the maximum number of iterations carried out (the default is 10). The iterations are terminated when the difference between two consecutive data estimates differs by less than `tol`%. The default value of `tol` is `1`.

Algorithm

For a given model, the missing data is estimated as parameters so as to minimize the output prediction errors obtained from the reconstructed data. See Section 14.2 in Ljung (1999). Treating missing outputs as parameters is not the best approach from a statistical point of view, but is a good approximation in many cases.

When no model is given, the algorithm alternates between estimating missing data and estimating models, based on the current reconstruction.

nkshift

Purpose

Shift data sequences

Syntax

```
Datas = nkshift(Data,nk)
```

Description

`Data` contains input-output data in the `iddata` format.

`nk` is a row vector with the same length as the number of input channels in `Data`.

`Datas` is an `iddata` object where the input channels in `Data` have been shifted according to `nk`. A positive value of `nk(ku)` means that input channel number `ku` is delayed `nk(ku)` samples.

`nkshift` supports both frequency- and time-domain data. For frequency-domain data it multiplies with $e^{ink\omega T}$ to obtain the same effect as shifting in the time domain. For continuous-time frequency-domain data (`Ts = 0`), `nk` should be interpreted as the shift in seconds.

`nkshift` lives in symbiosis with the `InputDelay` property of `idmodel`:

```
m1 = pem(dat,4,'InputDelay',nk)
```

is related to

```
m2 = pem(nkshift(dat,nk),4);
```

such that `m1` and `m2` are the same models, but `m1` stores the delay information for use when frequency responses, etc., are computed.

Note the difference from the `idss` and `idpoly` property `nk`.

```
m3 = pem(dat,4,'nk',nk)
```

gives a model that itself explicitly contains a delay of `nk` samples.

See Also

`idss`, `Algorithm Properties`

Purpose

Convert `idmodel` with noise channels to model with only measured channels

Syntax

```
mod1 = noisecnv(mod)
mod2 = noisecnv(mod,'norm')
```

Description

`mod` is any `idmodel`, `idarx`, `idgrey`, `idpoly`, or `idss`.

The noise input channels in `mod` are converted as follows: Consider a model with both measured input channels u (nu channels) and noise channels e (ny channels) with covariance matrix Λ

$$y = Gu + He$$
$$\mathrm{cov}(e) = \Lambda = LL'$$

where L is a lower triangular matrix. Note that `mod.NoiseVariance` = Λ. The model can also be described with unit variance, normalized noise source v:

$$y = Gu + HLv$$
$$\mathrm{cov}(v) = I$$

- `mod1 = noisecnv(mod)` converts the model to a representation of the system $[G\ H]$ with $nu+ny$ inputs and ny outputs. All inputs are treated as measured, and `mod1` does not have any noise model. The former noise input channels have names `e@yname`, where `yname` is the name of the corresponding output.
- `mod2 = noisecnv(mod,'norm')` converts the model to a representation of the system $[G\ HL]$ with $nu+ny$ inputs and ny outputs. All inputs are treated as measured, and `mod2` does not have any noise model. The former noise input channels have names `v@yname`, where `yname` is the name of the corresponding output. Note that the noise variance matrix factor L typically is uncertain (has a nonzero covariance). This is taken into account in the uncertainty description of `mod2`.
- If `mod` is a time series, that is, nu = 0, `mod1` is a model that describes the transfer function H with measured input channels. Analogously, `mod2` describes the transfer function HL.

Note the difference with subreferencing:

- `mod('m')` gives a description of G only.
- `mod('n')` gives a description of the noise model characteristics as a time-series model, that is, it describes *H* and also the covariance of *e*. In contrast, `noisecnv(m('n'))` describes just the transfer function *H*. To obtain a description of the normalized transfer function *HL*, use `noisecnv(m('n'),'norm')`

Converting the noise channels to measured inputs is useful to study the properties of the individual transfer functions from noise to output. It is also useful for transforming `idmodel` objects to representations that do not handle disturbance descriptions explicitly.

Purpose

Set step size for numerical differentiation

Syntax

```
nds = nuderst(pars)
```

Description

The function `pem` uses numerical differentiation with respect to the model parameters when applied to state-space structures. The same is true for many functions that transform model uncertainties to other representations.

The step size used in these numerical derivatives is determined by the M-file `nuderst`. The output argument `nds` is a row vector whose `k`th entry gives the increment to be used when differentiating with respect to the `k`th element of the parameter vector `pars`.

The default version of `nuderst` uses a very simple method. The step size is the maximum of 10^{-4} times the absolute value of the current parameter and 10^{-7}. You can adjust this to the actual value of the corresponding parameter by editing `nuderst`. Note that the nominal value, for example 0, of a parameter might not reflect its normal size.

nyquist

Purpose

Plot Nyquist curve of frequency function with confidence regions

Syntax

```
nyquist(m)
[fr,w] = nyquist(m)
[fr,w,covfr] = nyquist(m)
nyquist(m1,m2,m3,...,w)
nyquist(m1,'PlotStyle1',m2,'PlotStyle2',...)
nyquist(m1,m2,m3,..'sd*5',sd,'mode',mode)
```

Description

`nyquist` computes the complex-valued frequency response of `idmodel` and `idfrd` models. When invoked without left-hand arguments, `nyquist` produces a Nyquist plot on the screen, that is, a graph of the frequency response's imaginary part against its real part.

The argument `m` is an arbitrary `idmodel` or `idfrd` model. This model can be continuous or discrete, and SISO or MIMO. The `InputNames` and `OuputNames` of the models are used to plot the responses for different I/O channels in separate plots. Pressing the **Enter** key advances the plot from one input-output pair to the next one. You can select specific I/O channels with normal subreferencing: `m(ky,ku)`. With `mode = 'same'`, all plots are given in the same diagram.

`nyquist(m,w)` explicitly specifies the frequency range or frequency points to be used for the plot. To focus on a particular frequency interval `[wmin,wmax]`, set `w = {wmin,wmax}`. (Notice the curly brackets.) To use particular frequency points, set `w` to the vector of desired frequencies. Use `logspace` to generate logarithmically spaced frequency vectors. All frequencies should be specified in rad/s.

`nyquist(m1,m2,...,mN)` or `nyquist(m1,m2,...mN,w)` plots the Bode responses of several `idmodels` or `idfrd` models on a single figure. The models can be mixes of different sizes, and continuous or discrete. The sorting of the plots is based on the `InputNames` and `OutputNames`.

`nyquist(m1,'PlotStyle1',...,mN,'PlotStyleN')` further specifies which color, line style, and/or marker should be used to plot each system, as in

```
nyquist(m1,'r--',m2,'gx')
```

When `sd` is specified as a number larger than zero, confidence regions are also plotted. These are ellipses in the complex plane and correspond to the region

where the true response at the frequency in question is to be found with a confidence corresponding to `sd` standard deviations (of the Gaussian distribution).

If the argument indicating standard deviations is given as in `'sd+5'`, a confidence region is plotted for every 5:th frequency, marking the center point by `'+'`. The default is `'sd+10'`.

Note that the frequencies cannot be specified for `idfrd` objects. For those, the plot and response are calculated for the internally stored frequencies. If the frequencies `w` are specified when several models are treated, they will apply to all non-`idfrd` models in the list. If you want different frequencies for different models, you should first convert them to `idfrd` objects using the `idfrd` command.

For time-series models (no input channels), the Nyquist plot is not defined.

Arguments

When `nyquist` is called with a single system and output arguments,

```
fr = nyquist(m,w) or [fr,w,covfr] = nyquist(m)
```

no plot is drawn on the screen. If `m` has `ny` outputs and `nu` inputs, and `w` contains `Nw` frequencies, then `fr` is an `ny`-by-`nu`-by-`Nw` array such that `fr(ky,ku,k)` gives the complex-valued frequency response from input `ku` to output `ky` at the frequency `w(k)`. For a SISO model, use `fr(:)` to obtain a vector of the frequency response. The uncertainty information `covfr` is a 5-D array where `covfr(ky,ku,k,:,:))` is the 2-by-2 covariance matrix of the response from input `ku` to output `ky` at frequency `w(k)`. The 1,1 element is the variance of the real part, the 2,2 element is the variance of the imaginary part, and the 1,2 and 2,1 elements are the covariance between the real and imaginary parts.

`squeeze(covfr(ky,ku,k,:,:))` gives the covariance matrix of the corresponding response.

If `m` is a time series (no input), `fr` is returned as the (power) spectrum of the outputs, an `ny`-by-`ny`-by-`Nw` array. Hence `fr(:,:,k)` is the spectrum matrix at frequency `w(k)`. The element `fr(k1,k2,k)` is the cross spectrum between outputs `k1` and `k2` at frequency `w(k)`. When `k1 = k2`, this is the real-valued power spectrum of output `k1`. The `covfr` is then the covariance of the spectrum `fr`, so that `covfr(k1,k1,k)` is the variance of the power spectrum of output `k1` at frequency `w(k)`. No information about the variance of the cross spectra is normally given. (That is, `covfr(k1,k2,k) = 0` for `k1` not equal to `k2`.)

If the model `m` is not a time series, use `fr = nyquist(m('n'))` to obtain the spectrum information of the noise (output disturbance) signals.

Examples

```
g = spa(data)
m = n4sid(data,3)
nyquist(g,m,3)
```

See Also

`bode`, `etfe`, `ffplot`, `idfrd`, `spa`

Purpose

Estimate state-space models using subspace method

Syntax

```
m = n4sid(data)
m = n4sid(data,order,'Property1',Value1,...,'PropertyN',ValueN)
```

Description

The function `n4sid` estimates models in state-space form and returns them as an `idss` object `m`. It handles an arbitrary number of input and outputs, including the time-series case (no input). The state-space model is in the innovations form

$$x(t+Ts) = Ax(t) + Bu(t) + Ke(t)$$
$$y(t) = Cx(t) + Du(t) + e(t)$$

`m`: The resulting model as an `idss` object.

If `data` is continuous-time (frequency-domain) data, a corresponding continuous-time state-space model is estimated.

`data`: An `iddata` object containing the output-input data. Both time-domain and frequency-domain signals are supported. `data` can also be a `frd` or `idfrd` frequency-response data object.

`order`: The desired order of the state-space model. If `order` is entered as a row vector (as in `order = [1:10]`), preliminary calculations for all the indicated orders are carried out. A plot is then given that shows the relative importance of the dimension of the state vector. More precisely, the singular values of the Hankel matrices of the impulse response for different orders are graphed. You are prompted to select the order, based on this plot. The idea is to choose an order such that the singular values for higher orders are comparatively small. If `order = 'best'`, a model of "best" (default choice) order is computed among the orders 1:10. This is the default choice of `order`.

Estimating the D Matrix

Whether the D matrix is estimated or not is governed by the property `nk`, which is further described below. The default is that D is not estimated. By setting the `k`th entry of `nk` to 0, the `k`th column of D (corresponding to the `k`th input) is estimated. To estimate a full D matrix thus, let `nk = zeros(1,nu)` as in

```
m = n4sid(data,order,'nk',[0 .. 0])
```

This holds for both discrete- and continuous-time models.

Property Name/Property Value Pairs

The list of property name/property value pairs can contain any `idss` and algorithm properties. See `idss` and `Algorithm Properties`.

`idss` properties that are of particular interest for `n4sid` are

- `nk`: For time-domain data, this gives delays from the inputs to the outputs, a row vector with the same number of entries as the number of input channels. Default is `nk = [1 1... 1]`. Note that delays of 0 or 1 show up as zeros or estimated parameters in the D matrix. Delays larger than 1 mean that a special structure of the A, B, and C matrices is used to accommodate the delays. This also means that the actual order of the state-space model will be larger than `order`. For continuous-time models estimated from continuous-time (frequency-domain) data, the elements of `nk` are restricted to the values 1 and 0.
- `CovarianceMatrix` (can be abbreviated to `'co'`): Setting `CovarianceMatrix` to `'None'` blocks all calculations of uncertainty measures. These can take the major part of the computation time. Note that, for a `'Free'` parameterization, the individual matrix elements cannot be associated with any variance. (These parameters are not identifiable.) Instead, the resulting model `m` stores a hidden state-space model in canonical form that contains covariance information. This is used when the uncertainty of various input-output properties is calculated. It can also be retrieved by `m.ss = 'can'`. The actual covariance properties of `n4sid` estimates are not known today. Instead the Cramer-Rao bound is computed and stored as an indication of the uncertainty.
- `DisturbanceModel`: Setting `DisturbanceModel` to `'None'` will deliver a model with `K = 0`. This has no direct effect on the dynamics model other than that the default choice of `N4Horizon` will not involve past outputs.
- `InitialState`: The initial state is always estimated for better accuracy. However, it is returned with `m` only if `InitialState = 'Estimate'`.

Algorithm properties that are of special interest are

- `Focus`: Assumes the values `'Prediction'` (default), `'Simulation'`, `'Stability'`, passbands, or any SISO linear filter (given as an LTI or `idmodel` object, or as filter coefficients. See `Algorithm Properties`.) Setting `'Focus'` to `'Simulation'` chooses weights that should give a better simulation performance for the model. In particular, a stable model is

guaranteed. Selecting a linear filter focuses the fit to the frequency ranges that are emphasized by this filter.

- `N4Weight`: This property determines some weighting matrices used in the singular-value decomposition that is a central step in the algorithm. Two choices are offered: `'MOESP'`, corresponding to the MOESP algorithm by Verhaegen, and `'CVA'`, which is the canonical variable algorithm by Larimore. The default value is `'N4Weight' = 'Auto'`, which gives an automatic choice between the two options. `m.EstimationInfo.N4Weight` tells you what the actual choice turned out to be.
- `N4Horizon`: Determines the prediction horizons forward and backward used by the algorithm. This is a row vector with three elements: `N4Horizon = [r sy su]`, where `r` is the maximum forward prediction horizon. That is, the algorithm uses up to `r` step-ahead predictors. `sy` is the number of past outputs, and `su` is the number of past inputs that are used for the predictions. See pages 209 and 210 in Ljung (1999) for the exact meaning of this. These numbers can have a substantial influence on the quality of the resulting model, and there are no simple rules for choosing them. Making `'N4Horizon'` a k-by-3 matrix means that each row of `'N4Horizon'` is tried, and the value that gives the best (prediction) fit to data is selected. (This option cannot be combined with selection of model order.) If the property `'Trace'` is `'On'`, information about the results is given in the MATLAB Command Window.

  If you specify only one column in `'N4Horizon'`, the interpretation is `r=sy=su`. The default choice is `'N4Horizon' = 'Auto'`, which uses an Akaike Information Criterion (AIC) for the selection of `sy` and `su`. If `'DisturbanceModel' = 'None'`, `sy` is set to 0. Typing `m.EstimationInfor.N4Horizon` will tell you what the final choices of horizons were.

Algorithm

The variants of the implemented algorithm are described in Section 10.6 in Ljung (1999).

Examples

Build a fifth-order model from data with three inputs and two outputs. Try several choices of auxiliary orders. Look at the frequency response of the model.

```
z = iddata([y1 y2],[ u1 u2 u3]);
m = n4sid(z,5,'n4h',[7:15]','trace','on');
bode(m,'sd',3)
```

Estimate a continuous-time model, in a canonical form parameterization, focusing on the simulation behavior. Determine the order yourself after seeing the plot of singular values.

```
m = n4sid(m,[1:10],'foc','sim','ssp','can','ts',0)
bode(m)
```

References

vanOverschee, P., and B. DeMoor, *Subspace Identification of Linear Systems: Theory, Implementation, Applications*, Kluwer Academic Publishers, 1996.

Verhaegen, M., "Identification of the deterministic part of MIMO state space models," *Automatica,* Vol. 30, pp. 61-74, 1994.

Larimore, W.E., "Canonical variate analysis in identification, filtering and adaptive control," In *Proc. 29th IEEE Conference on Decision and Control*, pp. 596-604, Honolulu, 1990.

See Also

`idss`, `pem`, `Algorithm Properties`

Purpose

Estimate parameters of output-error model

Syntax

```
m = oe(data,orders)
m = oe(data,'nb',nb,'nf',nf,'nk',nk)
m = oe(data,orders,'Property1',Value1,'Property2',Value2,...)
```

Description

`oe` returns `m` as an `idpoly` object with the resulting parameter estimates, together with estimated covariances. The parameters of the output-error model structure

$$y(t) = \frac{B(q)}{F(q)}u(t-nk) + e(t)$$

are estimated using a prediction error method.

`data` is an `iddata` object containing the output-input data. Both time- and frequency-domain data are supported. Moreover, `data` can be an `frd` or `idfrd` frequency-response data object.

The structure information can either be given explicitly as

```
(...,'nb',nb,'nf',nf,'nk',nk,...)
```

or in the argument `orders`, given as

```
orders = [nb nf nk]
```

The parameters `nb` and `nf` are the orders of the output-error model and `nk` is the delay. Specifically,

$$nb: \qquad B(q) = b_1 + b_2 q^{-1} + \ldots + b_{nb} q^{-nb+1}$$

$$nf: \qquad F(q) = 1 + f_1 q^{-1} + \ldots + f_{nf} q^{-nf}$$

Alternatively, you can specify the vector as

```
orders = mi
```

where `mi` is an initial guess at the output-error model given in `idpoly` format. See “Polynomial Representation of Transfer Functions” on page 3-11.

For multiinput systems, nb, nf, and nk are row vectors with as many entries as there are input channels. Entry number i then describes the orders and delays associated with the ith input.

Continuous-Time Models

If data is continuous-time (frequency-domain) data, oe estimates a continuous-time model with transfer function

$$G(s) = \frac{B(s)}{F(s)} = \frac{b_{nb}s^{(nb-1)} + b_{nb-1}s^{(nb-2)} + \ldots + b_1}{s^{nf} + f_{nf}s^{(nf-1)} + \ldots + f_1}$$

The orders of the numerator and denominator are thus determined by nb and nf just as in the discrete-time case. However, the delay nk has no meaning and should be omitted. For multiinput systems, nb and nf are row vectors with obvious interpretation.

Properties

The structure and the estimation algorithm are affected by any property name/property value pairs that are set in the input argument list. Useful properties are 'Focus', 'InitialState', 'InputDelay', 'SearchDirection', 'MaxIter', 'Tolerance', 'LimitError', 'FixedParameter', and 'Trace'.

See Algorithm Properties, idpoly, and idmodel for details of these properties and their possible values.

oe does not support multioutput models. Use a state-space model for this case (see n4sid and pem).

Algorithm

oe uses essentially the same algorithm as armax, with modifications to the computation of prediction errors and gradients.

Examples

Suppose fast sampled data (Ts = 0.001) is available from a plant with a bandwidth of about 500 rad/s. The data is treated as continuous-time frequency-domain data, and a model of the type

$$G(s) = \frac{b}{s^3 + f_1 s^2 + f_2 s + f_3}$$

is estimated.

```
z = iddata(y,u,0.001);
zf = fft(z);
zf.ts = 0;
m = oe(zf,[1 3],'foc',[0 500])
```

See Also

armax, bj, idpoly, pem

pe

Purpose

Compute prediction errors associated with model and data set

Syntax

```
e = pe(m,data)
[e,x0] = pe(m,data,init)
```

Description

`data` is the output-input data set, given as an `iddata` object, and `m` is any `idmodel` object. Both time-domain and frequency-domain data are supported, and `data` can also be an `idfrd` object.

`e` is returned as an `iddata` object, so that `e.OutputData` contains the prediction errors that result when model `m` is applied to the data.

$$e(t) = H^{-1}(q)[y(t) - G(q)u(t)]$$

The argument `init` determines how to deal with the initial conditions:

- `init = 'e(stimate)'` means that the initial state is chosen so that the norm of prediction error is minimized. This initial state is returned as `x0`.
- `init = `d(elayexpand)'`: Same as 'estimate', but for a model with nonzero `InputDelay`, the delays are first converted to explicit model delays (using `inpd2nk`) so that they are contained in x0.
- `init = 'z(ero)'` sets the initial state to zero.
- `init = 'm(odel)'` uses the model's internally stored initial state.
- `init = x0i`, where `x0i` is a column vector of appropriate dimension, uses that value as initial state. For multiexperiment data, `x0i` may be a matrix whose columns give different initial states for each experiment. Notice that for a continuous-time model `m`, `x0` is the initial state for this model. Any modifications of the initial state that sampling might require are automatically handled. If `m` has a non-zero `InputDelay`, and you need to access the values of the inputs during this delay, you must first apply `inpd2nk(m)`. If `m` is continuous in time, it must first be sampled before `inpd2nk` can be applied.

If `init` is not specified, the model property `m.InitialState` is used, so that `'Estimate'`, `'Backcast'`, and `'Auto'` set `init = 'Estimate'`, while `m.InitialState = 'Zero'` sets `init = 'zero'`, and `'Fixed'` and `'Model'` set `init = 'model'`.

The output argument `x0` is the value of the initial state used. If `data` contains several experiments, `x0` is a matrix containing the initial states from each experiment.

See Also

`idmodel`, `resid`

pem

Purpose

Estimate parameters of general linear models

Syntax

```
m = pem(data)
m = pem(data,mi)
m = pem(data,mi,'Property1',Value1,...,'PropertyN',ValueN)
m = pem(data,orders)
m = pem(data,'P1D')
m = pem(data,'nx',ssorder)
m = pem(data,'na',na,'nb',nb,'nc',nc,'nd',nd,'nf',nf,'nk',nk)
m = pem(data,orders,'Property1',Value1,...,'PropertyN',ValueN)
```

Description

`pem` is the basic estimation command in the toolbox and covers a variety of situations.

`data` is always an `iddata` object that contains the input/output data. Both time-domain and frequency-domain signals are supported. `data` can also be an `frd` or `idfrd` frequency-response data object. Estimation of noise models (`K` in state-space models and `A`, `C`, and `D` in polynomial models) is not supported for frequency-domain data.

With Initial Model

`mi` is any `idmodel` object, `idarx`, `idpoly`, `idproc`, `idss`, or `idgrey`. It could be a result of another estimation routine, or constructed and modified by the constructors (`idarx`, `idpoly`, `idss`, `idgrey`, `idproc`) and `set`. The properties of `mi` can also be changed by any property name/property value pairs in `pem` as indicated in the syntax.

`m` is then returned as the best fitting model in the model structure defined by `mi`. The iterative search is initialized at the parameters of the initial/nominal model `mi`. `m` will be of the same class as `mi`.

Black-Box State-Space Models

With `m = pem(data,n)`, where `n` is a positive integer, or `m = pem(data,'nx',n)`, a state-space model of order `n` is estimated.

$$x(t+Ts) = Ax(t) + Bu(t) + Ke(t)$$
$$y(t) = Cx(t) + Du(t) + e(t)$$

If data is continuous-time (frequency-domain) data, a corresponding continuous-time state space model is estimated.

The default is that it is estimated in a 'Free' parameterization that can be further modified by the properties 'nk', 'DisturbanceModel', and 'InitialState' (see the reference pages for idss and n4sid). The model is initialized by n4sid and then further adjusted by optimizing the prediction error fit.

You can choose among several different orders by

```
m = pem(data,'nx',[n1,n2,...nN])
```

and you are then prompted for the "best" order. By

```
m = pem(data,'best')
```

an automatic choice of order among 1:10 is made.

```
m = pem(data)
```

is short for `m = pem(data,'best')`. To work with other delays, use, for example, `m = pem(data,'best','nk',[0,...0])`.

In this case m is returned as an idss model.

Estimating the D, K, and X0 Matrices

Whether the *D* matrix is estimated or not is governed by the property nk, which is further described below. The default is that *D* is not estimated. By setting the kth entry of nk to 0, the kth column of *D* (corresponding to the kth input) is estimated. To estimate a full *D* matrix, let `nk = zeros(1,nu)`, as in

```
m = pem(data,order,'nk',[0 .. 0])
```

This holds for both discrete- and continuous-time models.

For frequency-domain data, *K* is always fixed to 0. For time-domain data, *K* is estimated by default. To fix *K* to 0 in this case, use

```
m = pem(data,order,'DisturbanceModel','none')
```

Similarily, X0 is estimated if 'InitialState' is set to 'Estimate', and fixed to 0 if 'InitialState' is set to 'Zero'.

Black-Box Multiple-Input-Single-Output Models

The function pem also handles the general multiple-input-single-output structure

$$A(q)y(t) = \frac{B_1(q)}{F_1(q)}u_1(t-nk_1) + \ldots + \frac{B_{nu}(q)}{F_{nu}(q)}u_{nu}(t-nk_{nu}) + \frac{C(q)}{D(q)}e(t)$$

The orders of this general model are given either as

```
orders = [na nb nc nd nf nk]
```

or with `(...'na',na,'nb',nb,...)` as shown in the syntax. Here na, nb, nc, nd, and nf are the orders of the model, and nk is the delay(s). For multiinput systems, nb, nf, and nk are row vectors giving the orders and delays of each input. (See "Polynomial Representation of Transfer Functions" on page 3-11 for exact definitions of the orders.) When the orders are specified with separate entries, those not given are taken as zero.

For frequency-domain data, only estimation of B and F is supported. It is simpler to use oe in that case.

In this case m is returned as an idpoly object.

Continuous-Time Process Models

Entering for the initial model an acronym for a process model, as in

```
m = pem(data,'P2UI')
```

will estimate a continuous-time process model of the indicated type. See the reference page for Purpose for details of possible model types and associated property name/property value pairs.

In this case m is returned as an idproc model.

Properties

In all cases the algorithm is affected by the properties (see Algorithm Properties for details):

- Focus, with possible values 'Prediction' (default), 'Simulation', or a passband range.
- MaxIter and Tolerance govern the stopping criteria for the iterative search.

- `LimitError` deals with how the criterion can be made less sensitive to outliers and bad data.
- `MaxSize` determines the largest matrix ever formed by the algorithm. The algorithm goes into `for` loops to avoid larger matrices, which can be more efficient than using virtual memory.
- `Trace`, with possible values `'Off'`, `'On'`, and `'Full'`, governs the information sent to the MATLAB Command Window.

For black-box state-space models, `'N4Weight'` and `'N4Horizon'` will also affect the result, since these models are initialized with an `n4sid` estimate. See the reference page for `n4sid`.

Typical `idmodel` properties are (see `idmodel` properties for more details)

- `Ts` is the sampling interval. Set `'Ts' = 0` to obtain a continuous-time state-space model. For discrete-time models, `'Ts'` is automatically set to the sampling interval of the data. Note that, in the black-box case, it is usually better to first estimate a discrete-time model, and then convert that to continuous time using `d2c`.
- `nk` is the time delays from the inputs (not applicable to structured state-space models). Time delays specified by `'nk'` will be included in the model.
- `DisturbanceModel` determines the parameterization of `K` for free and canonical state-space parameterizations, as well as for `idgrey` models. It also determines whether a noise model should be included for `idproc` models.
- `InitialState`: The initial state can have a substantial influence on the estimation result for systems with slow responses. It is most pronounced for output-error models (`K = 0` for state-space and `na = nc = nd = 0` for input/output models). The default value `'Auto'` estimates the influence of the initial state and sets the value to `'Estimate'`, `'Backcast'`, or `'Zero'` based on this effect. Possible values of `'InitialState'` are `'Auto'`, `'Estimate'`, `'Backcast'`, `'Zero'`, and `'Fixed'`. See “Initial State” on page 3-100.

Examples

Here is an example of a system with three inputs and two outputs. A canonical form state-space model of order 5 is sought.

```
z = iddata([y1 y2],[ u1 u2 u3]);
m = pem(z,5,'ss','can')
```

Building an ARMAX model for the response to output 2,

```
ma = pem(z(:,2,:),'na',2,'nb',[2 3 1],'nc',2,'nk',[1 2 0])
```

Comparing the models (`compare` automatically matches the channels using the channel names),

```
compare(z,m,ma)
```

Algorithm

`pem` uses essentially the same algorithm as `armax`, with modifications to the computation of prediction errors and gradients.

See Also

`armax`, `bj`, `oe`, `idss`, `idpoly`, `idgrey`, `idmodel`, `Algorithm Properties`, `EstimationInfo`

Purpose

Determine level of excitation of input signals

Syntax

```
Ped = pexcit(Data)
[Ped.Maxnr] = pexcit(Data,Maxnr,Threshold)
```

Description

`Data` is an `iddata` object with time- or frequency-domain signals.

`Ped` is the degree or order of excitation of the inputs in `Data`. A row vector of integers with as many components as there are inputs in `Data`. The intuitive interpretation of the degree of excitation in an input is the order of a model that the input is capable of estimating in an unambiguous way.

`Maxnr` is the maximum order tested. Default is `min(N/3,50)`, where `N` is the number of input data.

`Threshold` is the threshold level used to measure which singular values are significant. Default is `1e-9`.

References

Section 13.2 in Ljung (1999).

See Also

`iddata`, `advice`

plot (iddata)

Purpose

Plot input-output `iddata`

Syntax

```
plot(data)
plot(d1,...,dN)
plot(d1,PlotStyle1,...,dN,PlotStyleN)
```

Description

`data` is the output-input data to be graphed, given as an `iddata` object. A split plot is obtained with the outputs on top and the inputs at the bottom.

One plot for each I/O channel combination is produced. Pressing the **Enter** key advances the plot. Typing **Ctrl+C** aborts the plotting in an orderly fashion.

To plot a specific interval, use `plot(data(200:300))`. To plot specific input/output channels, use `plot(data(:,ky,ku))`, consistent with the subreferencing of `iddata` objects (see `iddata`).

If `data.intersample = 'zoh'`, the input is piecewise constant between sampling points, and it is then graphed accordingly.

To plot several `iddata` sets `d1,...,dN`, use `plot(d1,...,dN)`. I/O channels with the same experiment name, input name, and output name are always plotted in the same plot.

With `PlotStyle`, the color, line style, and marker of each data set can be specified

```
plot(d1,'y:*',d2,'b')
```

just as in the regular `plot` command.

See Also

`iddata`

Purpose

Plot `idmodel` properties using LTI viewer in Control Systems Toolbox

Syntax

See `view`.

polydata

Purpose

Convert model to input-output polynomials

Syntax

```
[A,B,C,D,F] = polydata(m)
[A,B,C,D,F,dA,dB,dC,dD,dF] = polydata(m)
```

Description

This is essentially the inverse of the `idpoly` constructor. It returns the polynomials of the general model

$$A(q)y(t) = \frac{B_1(q)}{F_1(q)}u_1(t-nk_1) + \ldots + \frac{B_{nu}(q)}{F_{nu}(q)}u_{nu}(t-nk_{nu}) + \frac{C(q)}{D(q)}e(t)$$

as contained in the model `m`.

`dA`, `dB`, etc. are the standard deviations of `A`, `B`, etc.

`m` can be any single-output `idmodel`, that is, not just `idpoly`. For multioutput models you can use `[A,B,C,D,F] = polydata(m(ky,:))` to obtain the polynomials for the `ky`th output.

See Also

`idmodel`, `idpoly`, `tfdata`

Purpose

Predict output k steps ahead

Syntax

```
yp = predict(m,data)
[yp,x0p,mpred] = predict(m,data,k,'InitialState',init)
```

Description

`data` is the output-input data as an `iddata` object, and `m` is any `idmodel` object (`idpoly`, `idproc`, `idss`, `idgrey`, or `idarx`). `predict` is meaningful only for time-domain data.

The argument `k` indicates that the k step-ahead prediction of `y` according to the model `m` is computed. In the calculation of $yp(t)$, the model can use outputs up to time

$$t-k: y(s), s = t-k, t-k-1, \ldots$$

and inputs up to the current time t. The default value of `k` is `1`.

The output `yp` is an `iddata` object containing the predicted values as `OutputData`.

`x0p` is the used (estimated) initial state vector. For multiexperiment data, `x0p` is a matrix, whose columns contain the initial states for each experiment.

The output argument `mpred` contains the k step-ahead predictor. This is given as a cell array, whose kth entry is an `idpoly` model for the predictor of output number k. Note that these predictor models have as input both input and output signals in the data set. The channel names indicate how the predictor model and the data fit together.

`init` determines how to deal with the initial state:

- `init ='e(stimate)'`: The initial state is set to a value that minimizes the norm of the prediction error associated with the model and the data.
- `init = `d(elayexpand)'`: Same as 'estimate', but for a model with nonzero `InputDelay`, the delays are first converted to explicit model delays (using `inpd2nk`) so that they are contained in `x0p`.
- `init = 'z(ero)'` sets the initial state to zero.
- `init = 'm(odel)'` uses the model's internally stored initial state.
- `init = x0`, where `x0` is a column vector of appropriate dimension, uses that value as initial state. For multiexperiment data, `x0` can be a matrix whose columns give different initial states for each experiment. Notice that for a

continuous-time model `m`, `x0` is the initial state for this model. Any modifications of the initial state that sampling might require are automatically handled. If `m` has a non-zero `InputDelay`, and you need to access the values of the inputs during this delay, you must first apply `inpd2nk(m)`. When `m` is a continuous-time model, it must first be sampled before `inpd2nk` can be applied.

If `init` is not specified, the model property `m.InitialState` is used, so that `'Estimate'`, `'Backcast'`, and `'Auto'` set `init = 'Estimate'`, while `m.InitialState = 'Zero'` sets `init = 'zero'`, and `'Model'` and `'Fixed'` set `init = 'model'`.

An important use of `predict` is to evaluate a model's properties in the mid-frequency range. Simulation with `sim` (which conceptually corresponds to `k = inf`) can lead to levels that drift apart, since the low-frequency behavior is emphasized. One step-ahead prediction is not a powerful test of the model's properties, since the high-frequency behavior is stressed. The trivial predictor $\hat{y}(t) = y(t-1)$ can give good predictions in case the sampling of the data is fast.

Another important use of `predict` is to evaluate time-series models. The natural way of studying a time-series model's ability to reproduce observations is to compare its k step-ahead predictions with actual data.

Note that for output-error models, there is no difference between the k step-ahead predictions and the simulated output, since, by definition, output-error models only use past inputs to predict future outputs.

Algorithm

The model is evaluated in state-space form, and the state equations are simulated k steps ahead with initial value $x(t-k) = \hat{x}(t-k)$, where $\hat{x}(t-k)$ is the Kalman filter state estimate.

Examples

Simulate a time series, estimate a model based on the first half of the data, and evaluate the four step-ahead predictions on the second half.

```
m0 = idpoly([1 -0.99],[],[1 -1 0.2]);
e = iddata([],randn(400,1));
y = sim(m0,e);
m = armax(y(1:200),[1 2]);
yp = predict(m,y,4);
plot(y(201:400),yp(201:400))
```

Note that the last two commands are also achieved by

```
compare(y,m,4,201:400);
```

See Also

compare, sim, pe

present

Purpose

Display information in `idmodel` model, including uncertainty

Syntax

```
present(m)
```

Description

The `present` function displays the model `m`, together with the estimated standard deviations of the parameters, loss function, and Akaike's Final Prediction Error (FPE) Criterion (which essentially equals the AIC). It also displays information about how `m` was created.

`present` thus gives more detailed information about the model than the standard `display` function.

Purpose

Plot zeros and poles with confidence regions

Syntax

```
pzmap(m)
pzmap(m,'sd',sd)
pzmap(m1,m2,m3,...)
pzmap(m1,'PlotStyle1',m2,'PlotStyle2',...,'sd',sd)
pzmap(m1,m2,m3,..,'sd',sd,'mode',mode,'axis',axis)
```

Description

`m` is any `idmodel` object: `idarx`, `idgrey`, `idss`, `idproc`, or `idpoly`.

The zeros and poles of `m` are graphed, with `o` denoting zeros and `x` denoting poles. Poles and zeros at infinity are ignored. For discrete-time models, zeros and poles at the origin are also ignored.

The Property/Value pairs `` `sd'/sd, `mode'/mode `` and `` `axis'/axis `` can appear in any order. They are explained below.

If `sd` has a value larger than zero, confidence regions around the poles and zeros are also graphed. The regions corresponding to `sd` standard deviations are marked. The default value is `sd = 0`. Note that the confidence regions might sometimes stretch outside the plot, but they are always symmetric around the indicated zero or pole.

If the poles and zeros are associated with a discrete-time model, a unit circle is also drawn. For continuous-time models, the real and imaginary axes are drawn.

When `mi` contains information about several different input/output channels, you have the following options:

`mode = 'sub'` splits the screen into several plots, one for each input/output channel. These are based on the `InputName` and `OutputName` properties associated with the different models.

`mode = 'same'` gives all plots in the same diagram. Pressing the **Enter** key advances the plots.

`mode = 'sep'` erases the previous plot before the next channel pair is treated.

The default value is `mode = 'sub'`.

`axis = [x1 x2 y1 y2]` fixes the axis scaling accordingly. `axis = s` is the same as

```
axis = [-s s -s s]
```

You can select the colors associated with the different models by using the argument `PlotStyle`. Use `PlotStyle = 'b'`, `'g'`, etc. Markers and line styles are not used.

The noise input channels in `m` are treated as follows: Consider a model `m` with both measured input channels u (nu channels) and noise channels e (ny channels) with covariance matrix Λ

$$y = Gu + He$$
$$\mathrm{cov}(e) = \Lambda = LL'$$

where L is a lower triangular matrix. Note that `m.NoiseVariance` = Λ. The model can also be described with a unit variance, normalized noise source v.

$$y = Gu + HLv$$
$$\mathrm{cov}(v) = I$$

Then,

- `pzmap(m)` plots the zeros and poles of the transfer function G.
- `pzmap(m('n'))` plots the zeros and poles of the transfer function H (ny inputs and ny outputs). The input channels have names `e@yname`, where `yname` is the name of the corresponding output.
- If `m` is a time series, that is nu = `0`, `pzmap(m)` plots the zeros and poles of the transfer function H.
- `pzmap(noisecnv(m))` plots the zeros and poles of the transfer function [G H] (nu+ny inputs and ny outputs). The noise input channels have names `e@yname`, where `yname` is the name of the corresponding output.
- `pzmap(noisecnv(m,'norm'))` plots the zeros and poles of the transfer function [G HL] (nu+ny inputs and ny outputs). The noise input channels have names `v@yname`, where `yname` is the name of the corresponding output.

Examples

```
mbj = bj(data,[2 2 1 1 1]);
mar = armax(data,[2 2 2 1]);
pzmap(mbj,mar,'sd',3)
```

shows all zeros and poles of two models along with the confidence regions corresponding to three standard deviations.

See Also idmodel, zpkdata

Purpose

Estimate recursively parameters of ARMAX or ARMA model

Syntax

```
thm = rarmax(z,nn,adm,adg)
[thm,yhat,P,phi,psi] = rarmax(z,nn,adm,adg,th0,P0,phi0,psi0)
```

Description

The parameters of the ARMAX model structure

$$A(q)y(t) = B(q)u(t-nk) + C(q)e(t)$$

are estimated using a recursive prediction error method.

The input-output data is contained in `z`, which is either an `iddata` object or a matrix `z = [y u]` where `y` and `u` are column vectors. `nn` is given as

```
nn = [na nb nc nk]
```

where `na`, `nb`, and `nc` are the orders of the ARMAX model, and `nk` is the delay. Specifically,

$$na: \quad A(q) = 1 + a_1 q^{-1} + \ldots + a_{na} q^{-na}$$

$$nb: \quad B(q) = b_1 + b_2 q^{-1} + \ldots + b_{nb} q^{-nb+1}$$

$$nc: \quad C(q) = 1 + c_1 q^{-1} + \ldots + c_{nc} q^{-nc}$$

See “Polynomial Representation of Transfer Functions” on page 3-11 for more information.

If `z` represents a time series `y` and `nn = [na nc]`, `rarmax` estimates the parameters of an ARMA model for `y`.

$$A(q)y(t) = C(q)e(t)$$

Only single-input, single-output models are handled by `rarmax`. Use `rpem` for the multiinput case.

The estimated parameters are returned in the matrix `thm`. The `k`th row of `thm` contains the parameters associated with time `k`; that is, they are based on the data in the rows up to and including row `k` in `z`. Each row of `thm` contains the estimated parameters in the following order:

```
thm(k,:) = [a1,a2,...,ana,b1,...,bnb,c1,...,cnc]
```

`yhat` is the predicted value of the output, according to the current model; that is, row k of `yhat` contains the predicted value of `y(k)` based on all past data.

The actual algorithm is selected with the two arguments `adm` and `adg`. These are described under `rarx`.

The input argument `th0` contains the initial value of the parameters, a row vector consistent with the rows of `thm`. The default value of `th0` is all zeros.

The arguments `P0` and `P` are the initial and final values, respectively, of the scaled covariance matrix of the parameters. See `rarx`. The default value of `P0` is 10^4 times the unit matrix. The arguments `phi0`, `psi0`, `phi`, and `psi` contain initial and final values of the data vector and the gradient vector, respectively. The sizes of these depend on the chosen model orders. The normal choice of `phi0` and `psi0` is to use the outputs from a previous call to `rarmax` with the same model orders. (This call could be a dummy call with default input arguments.) The default values of `phi0` and `psi0` are all zeros.

Note that the function requires that the delay `nk` be larger than `0`. If you want `nk = 0`, shift the input sequence appropriately and use `nk = 1`.

Algorithm

The general recursive prediction error algorithm (11.44), (11.47) through (11.49) of Ljung (1999) is implemented. See “Recursive Parameter Estimation” on page 3-86 for more information.

Examples

Compute and plot, as functions of time, the four parameters in a second-order ARMA model of a time series given in the vector `y`. The forgetting factor algorithm with a forgetting factor of 0.98 is applied.

```
thm = rarmax(y,[2 2],'ff',0.98);
plot(thm)
```

Purpose

Estimate recursively parameters of ARX or AR model

Syntax

```
thm = rarx(z,nn,adm,adg)
[thm,yhat,P,phi] = rarx(z,nn,adm,adg,th0,P0,phi0)
```

Description

The parameters of the ARX model structure

$$A(q)y(t) = B(q)u(t-nk) + e(t)$$

are estimated using different variants of the recursive least squares method.

The input-output data is contained in `z`, which is either an `iddata` object or a matrix `z = [y u]` where `y` and `u` are column vectors. `nn` is given as

```
nn = [na nb nk]
```

where `na` and `nb` are the orders of the ARX model, and `nk` is the delay. Specifically,

$$na: \qquad A(q) = 1 + a_1 q^{-1} + \ldots + a_{na} q^{-na}$$

$$nb: \qquad B(q) = b_1 + b_2 q^{-1} + \ldots + b_{nb} q^{-nb+1}$$

See (Equation 3-13) in Chapter 3, "Tutorial," for more information.

If `z` is a time series `y` and `nn = na`, `rarx` estimates the parameters of an AR model for `y`.

$$A(q)y(t) = e(t)$$

Models with several inputs

$$A(q)y(t) = B_1(q)u_1(t-nk_1) + \ldots B_{nu}u_{nu}(t-nk_{nu}) + e(t)$$

are handled by allowing `u` to contain each input as a column vector,

```
u = [u1 ... unu]
```

and by allowing `nb` and `nk` to be row vectors defining the orders and delays associated with each input.

Only single-output models are handled by `rarx`.

The estimated parameters are returned in the matrix `thm`. The `k`th row of `thm` contains the parameters associated with time `k`; that is, they are based on the data in the rows up to and including row `k` in `z`. Each row of `thm` contains the estimated parameters in the following order.

```
thm(k,:) = [a1,a2,...,ana,b1,...,bnb]
```

In the case of a multiinput model, all the *b* parameters associated with input number 1 are given first, and then all the *b* parameters associated with input number 2, and so on.

`yhat` is the predicted value of the output, according to the current model; that is, row `k` of `yhat` contains the predicted value of `y(k)` based on all past data.

The actual algorithm is selected with the two arguments `adg` and `adm`. These are described in "Recursive Parameter Estimation" on page 3-86. The options are as follows:

- With `adm = 'ff'` and `adg = lam` the *forgetting factor* algorithm (Equation 3-65abd) and (Equation 3-67) is obtained with forgetting factor λ = `lam`. This is what is often referred to as recursive least squares (RLS). In this case the matrix `P` has the following interpretation: R_2`/2 * P` is approximately equal to the covariance matrix of the estimated parameters. Here R_2 is the variance of the innovations (the true prediction errors $e(t)$ in (Equation 3-62).
- With `adm ='ug'` and `adg = gam`, the *unnormalized gradient* algorithm (Equation 3-65abc) and (Equation 3-68) is obtained with gain *gamma* = `gam`. This algorithm is commonly known as normalized least mean squares (LMS).
- Similarly, `adm ='ng'` and `adg = gam` give the *normalized gradient* or normalized least mean squares (NLMS) algorithm (Equation 3-65abc) and (Equation 3-69). In these cases, `P` is not defined or applicable.
- With `adm ='kf'` and `adg = R1`, the *Kalman filter based* algorithm (Equation 3-65) is obtained with R_2= `1` and R_1 = `R1`. If the variance of the innovations $e(t)$ is not unity but R_2; then R_2 `* P` is the covariance matrix of the parameter estimates, while R_1 = `R1 /` R_2 is the covariance matrix of the parameter changes in (Equation 3-63).
- The input argument `th0` contains the initial value of the parameters, a row vector consistent with the rows of `thm`. The default value of `th0` is all zeros.

- The arguments `P0` and `P` are the initial and final values, respectively, of the scaled covariance matrix of the parameters. The default value of `P0` is 10^4 times the identity matrix.
- The arguments `phi0` and `phi` contain initial and final values, respectively, of the data vector.

$$\varphi(t) = [y(t-1), \ldots, y(t-na), u(t-1), \ldots u(t-nb-nk+1)]$$

Then, if

```
z = [y(1),u(1); ... ;y(N),u(N)]
```

you have `phi0` = $\varphi(1)$ and `phi` = $\varphi(N)$. The default value of `phi0` is all zeros. For online use of `rarx`, use `phi0`, `th0`, and `P0` as the previous outputs `phi`, `thm` (last row), and `P`.

Note that the function requires that the delay `nk` be larger than `0`. If you want `nk = 0`, shift the input sequence appropriately and use `nk = 1`. See `nkshift`.

Examples

Adaptive noise canceling: The signal y contains a component that has its origin in a known signal r. Remove this component by estimating, recursively, the system that relates r to y using a sixth-order FIR model together with the NLMS algorithm.

```
z = [y r];
[thm,noise] = rarx(z,[0 6 1],'ng',0.1);
% noise is the adaptive estimate of the noise
% component of y
plot(y-noise)
```

If the above application is a true online one, so that you want to plot the best estimate of the signal `y - noise` at the same time as the data y and u become available, proceed as follows.

```
phi = zeros(6,1); P=1000*eye(6);
th = zeros(1,6); axis([0 100 -2 2]);
plot(0,0,'*'), hold on
% The loop:
while ~abort
[y,r,abort] = readAD(time);
[th,ns,P,phi] = rarx([y r],'ff',0.98,th,P,phi);
plot(time,y-ns,'*')
```

```
time = time +Dt
end
```

This example uses a forgetting factor algorithm with a forgetting factor of 0.98. `readAD` represents an M-file that reads the value of an A/D converter at the indicated time instant.

rbj

Purpose

Estimate recursively parameters of Box-Jenkins model

Syntax

```
thm = rbj(z,nn,adm,adg)
[thm,yhat,P,phi,psi] = ... rbj(z,nn,adm,adg,th0,P0,phi0,psi0)
```

Description

The parameters of the Box-Jenkins model structure

$$y(t) = \frac{B(q)}{F(q)}u(t-nk) + \frac{C(q)}{D(q)}e(t)$$

are estimated using a recursive prediction error method.

The input-output data is contained in `z`, which is either an `iddata` object or a matrix `z = [y u]` where `y` and `u` are column vectors. `nn` is given as

```
nn = [nb nc nd nf nk]
```

where `nb`, `nc`, `nd`, and `nf` are the orders of the Box-Jenkins model, and `nk` is the delay. Specifically,

nb: $B(q) = b_1 + b_2 q^{-1} + \ldots + b_{nb} q^{-nb+1}$

nc: $C(q) = 1 + c_1 q^{-1} + \ldots + c_{nc} q^{-nc}$

nd: $D(q) = 1 + d_1 q^{-1} + \ldots + d_{nd} q^{-nd}$

nf: $F(q) = 1 + f_1 q^{-1} + \ldots + f_{nf} q^{-nf}$

See “Polynomial Representation of Transfer Functions” on page 3-11 for more information.

Only single-input, single-output models are handled by `rbj`. Use `rpem` for the multiinput case.

The estimated parameters are returned in the matrix `thm`. The `k`th row of `thm` contains the parameters associated with time `k`; that is, they are based on the data in the rows up to and including row `k` in `z`. Each row of `thm` contains the estimated parameters in the following order.

```
thm(k,:) = [b1,...,bnb,c1,...,cnc,d1,...,dnd,f1,...,fnf]
```

`yhat` is the predicted value of the output, according to the current model; that is, row `k` of `yhat` contains the predicted value of `y(k)` based on all past data.

The actual algorithm is selected with the two arguments `adm` and `adg`. These are described under `rarx`.

The input argument `th0` contains the initial value of the parameters, a row vector consistent with the rows of `thm`. The default value of `th0` is all zeros.

The arguments `P0` and `P` are the initial and final values, respectively, of the scaled covariance matrix of the parameters. See `rarx`. The default value of `P0` is 10^4 times the unit matrix. The arguments `phi0`, `psi0`, `phi`, and `psi` contain initial and final values of the data vector and the gradient vector, respectively. The sizes of these depend on the chosen model orders. The normal choice of `phi0` and `psi0` is to use the outputs from a previous call to `rbj` with the same model orders. (This call could be a dummy call with default input arguments.) The default values of `phi0` and `psi0` are all zeros.

Note that the function requires that the delay `nk` be larger than `0`. If you want `nk = 0`, shift the input sequence appropriately and use `nk = 1`.

Algorithm

The general recursive prediction error algorithm (11.44) of Ljung (1900) is implemented. See also “Recursive Parameter Estimation” on page 3-86.

realdata

Purpose

Determine whether `iddata` is based on real-valued signals

Syntax

```
realdata(data)
```

Description

`realdata` returns 1 if

- `data` contains only real-valued signals.
- `data` contains frequency-domain signals, obtained by Fourier transformation of real-valued signals.

Otherwise `realdata` returns `0`.

Notice the difference with `isreal`:

```
load iddata1
isreal(z1); % returns 1
zf = fft(z1);
isreal(zf) % returns 0
realdata(zf) % returns 1
zf = complex(zf) % adds negative frequencies to zf
realdata(zf) % still returns 1
```

Purpose

Resample data by interpolation and decimation

Syntax

```
datar = resample(data,P,Q)
datar = resample(data,P,Q,,filter_order)
```

Description

`data`: The data to be resampled, given as an `iddata` object

`datar`: The resampled data returned as an `iddata` object

`P`, `Q`: Integers that determine the resampling factor. The new sampling interval will be `Q/P` times the original one, so `resample(z,1,Q)` means decimation with a factor `Q`.

`filter_order`: Determines the order of the presampling filters used before interpolation and decimation. Default is `10`.

Algorithm

If the Signal Processing Toolbox is available, the resampling is achieved by calls to the `resample` function in that toolbox. The intersample character of the input, as described by `data.InterSample`, is taken into account.

Otherwise, use the function `datar = idresamp(data,R)`, where `R=Q/P`. Then the data is interpolated by a factor `P` and then decimated by a factor `Q`. The interpolation and decimation are preceded by prefiltering, and follow the same algorithms as in the routines `interp` and `decimate` in the Signal Processing Toolbox.

Examples

Resample by increasing the sampling rate by a factor of 1.5 and compare the signals.

```
plot(u)
ur = resample(u,3,2);
plot(u,ur)
```

resid

Purpose

Compute and test model residuals (prediction errors)

Syntax

```
resid(m,data)
resid(m,data,Type)
resid(m,data,Type,M)
e = resid(m,data);
```

Description

`data` contains the output-input data as an `iddata` object. Both time-domain and frequency-domain data are supported. `data` can also be an `idfrd` object.

`m` is the model to be evaluated on the given data set. It is any `idmodel` object.

In all cases the residuals *e* associated with the data and the model are computed. This is done as in the command `pe` with a default choice of `init`.

When called without output arguments, `resid` produces a plot. The plot can be of three kinds depending on the argument `Type`:

- `Type = 'Corr'` (only available for time-domain data): The autocorrelation function of `e` and the cross correlation between `e` and the input(s) `u` are computed and displayed. The 99% confidence intervals for these values are also computed and shown as a yellow region. The computation of the confidence region is done assuming `e` to be white and independent of `u`. The functions are displayed up to lag `M`, which is 25 by default.
- `Type = 'ir'`: The impulse response (up to lag `M`, which is 25 by default) from the input to the residuals is plotted with a 99% confidence region around zero marked as a yellow area. Negative lags up to `M/4` are also included to investigate feedback effects. (The result is the same as `impulse(e,'sd',2.58,'fill',M)`.)
- `Type = 'fr'`: The frequency response from the input to the residuals (based on a high-order FIR model) is shown as a Bode plot. A 99% confidence region around zero is also marked as a yellow area.

The default for time-domain data is `Type = 'Corr'`. For frequency-domain data, the default is `Type = 'fr'`.

With an output argument, no plot is produced, and `e` is returned with the residuals (prediction errors) associated with the model and the data. It is an `iddata` object with the residuals as outputs and the input in `data` as inputs. That means that `e` can be directly used to build model error models, that is,

models that describe the dynamics from the input to the residuals (which should be negligible if `m` is a good description of the system).

See "Model Structure Selection and Validation" on page 3-70 for more information.

Examples

Here are some typical model validation commands.

```
e = resid(m,data);
plot(e)
compare(data,m);
```

To compute a model error model, that is, a model to input to the residuals to see if any essential unmodeled dynamics are left, do the following:

```
e = resid(m,data);
me = arx(e,[10 10 0]);
bode(me,'sd',3,fill')
```

References

Ljung (1999), Section 16.6.

See Also

`compare`, `idgrey`, `idarx`, `idpoly`, `idproc`, `idss`, `pem`

Purpose

Estimate output-error models (IIR-filters) recursively

Syntax

```
thm = roe(z,nn,adm,adg)
[thm,yhat,P,phi,psi] = roe(z,nn,adm,adg,th0,P0,phi0,psi0)
```

Description

The parameters of the output-error model structure

$$y(t) = \frac{B(q)}{F(q)}u(t-nk) + e(t)$$

are estimated using a recursive prediction error method.

The input-output data are contained in `z`, which is either an `iddata` object or a matrix `z = [y u]` where `y` and `u` are column vectors. `nn` is given as

```
nn = [nb nf nk]
```

where `nb` and `nf` are the orders of the output-error model, and `nk` is the delay. Specifically,

$$nb: \qquad B(q) = b_1 + b_2 q^{-1} + \ldots + b_{nb} q^{-nb+1}$$

$$nf: \qquad F(q) = 1 + f_1 q^{-1} + \ldots + f_{nf} q^{-nf}$$

See “Polynomial Representation of Transfer Functions” on page 3-11 for more information.

Only single-input, single-output models are handled by `roe`. Use `rpem` for the multiinput case.

The estimated parameters are returned in the matrix `thm`. The `k`th row of `thm` contains the parameters associated with time `k`; that is, they are based on the data in the rows up to and including row `k` in `z`.

Each row of `thm` contains the estimated parameters in the following order.

```
thm(k,:) = [b1,...,bnb,f1,...,fnf]
```

`yhat` is the predicted value of the output, according to the current model; that is, row `k` of `yhat` contains the predicted value of `y(k)` based on all past data.

The actual algorithm is selected with the two arguments `adg` and `adm`. These are described under `rarx`.

The input argument `th0` contains the initial value of the parameters, a row vector consistent with the rows of `thm`. The default value of `th0` is all zeros.

The arguments `P0` and `P` are the initial and final values, respectively, of the scaled covariance matrix of the parameters. See `rarx`. The default value of `P0` is 10^4 times the unit matrix. The arguments `phi0`, `psi0`, `phi`, and `psi` contain initial and final values of the data vector and the gradient vector, respectively. The sizes of these depend on the chosen model orders. The normal choice of `phi0` and `psi0` is to use the outputs from a previous call to `roe` with the same model orders. (This call could be a dummy call with default input arguments.) The default values of `phi0` and `psi0` are all zeros.

Note that the function requires that the delay `nk` be larger than `0`. If you want `nk = 0`, shift the input sequence appropriately and use `nk = 1`.

Algorithm

The general recursive prediction error algorithm (11.44) of Ljung (1999) is implemented. See also "Recursive Parameter Estimation" on page 3-86.

See Also

`oe`, `rarx`, `rbj`, `rplr`, `rpem`, `nkshift`

rpem

Purpose

Estimate general input-output models using recursive prediction error method

Syntax

```
thm = rpem(z,nn,adm,adg)
[thm,yhat,P,phi,psi] = rpem(z,nn,adm,adg,th0,P0,phi0,psi0)
```

Description

The parameters of the general linear model structure

$$A(q)y(t) = \frac{B_1(q)}{F_1(q)}u_1(t-nk_1) + \ldots + \frac{B_{nu}(q)}{F_{nu}(q)}u_{nu}(t-nk_{nu}) + \frac{C(q)}{D(q)}e(t)$$

are estimated using a recursive prediction error method.

The input-output data is contained in `z`, which is either an `iddata` object or a matrix `z = [y u]` where `y` and `u` are column vectors. (In the multiinput case, `u` contains one column for each input.) `nn` is given as

```
nn = [na nb nc nd nf nk]
```

where `na`, `nb`, `nc`, `nd`, and `nf` are the orders of the model, and `nk` is the delay. For multiinput systems, `nb`, `nf`, and `nk` are row vectors giving the orders and delays of each input. See "Polynomial Representation of Transfer Functions" on page 3-11 for an exact definition of the orders.

The estimated parameters are returned in the matrix `thm`. The `k`th row of `thm` contains the parameters associated with time `k`; that is, they are based on the data in the rows up to and including row `k` in `z`. Each row of `thm` contains the estimated parameters in the following order.

```
thm(k,:) = [a1,a2,...,ana,b1,...,bnb,...
  c1,...,cnc,d1,...,dnd,f1,...,fnf]
```

For multiinput systems, the B part in the above expression is repeated for each input before the C part begins, and the F part is also repeated for each input. This is the same ordering as in `m.par`.

`yhat` is the predicted value of the output, according to the current model; that is, row `k` of `yhat` contains the predicted value of `y(k)` based on all past data.

The actual algorithm is selected with the two arguments `adg` and `adm`. These are described under `rarx`.

The input argument `th0` contains the initial value of the parameters, a row vector consistent with the rows of `thm`. The default value of `th0` is all zeros.

The arguments `P0` and `P` are the initial and final values, respectively, of the scaled covariance matrix of the parameters. See `rarx`. The default value of `P0` is 10^4 times the unit matrix. The arguments `phi0`, `psi0`, `phi`, and `psi` contain initial and final values of the data vector and the gradient vector, respectively. The sizes of these depend on the chosen model orders. The normal choice of `phi0` and `psi0` is to use the outputs from a previous call to `rpem` with the same model orders. (This call could be a dummy call with default input arguments.) The default values of `phi0` and `psi0` are all zeros.

Note that the function requires that the delay `nk` be larger than `0`. If you want `nk = 0`, shift the input sequence appropriately and use `nk = 1`.

Algorithm

The general recursive prediction error algorithm (11.44) of Ljung (1999) is implemented. See also "Recursive Parameter Estimation" on page 3-86.

For the special cases of ARX/AR models, and of single-input ARMAX/ARMA, Box-Jenkins, and output-error models, it is more efficient to use `rarx`, `rarmax`, `rbj`, and `roe`.

See Also

`pem`, `rarmax`, `rarx`, `rbj`, `roe`, `rplr`, `nkshift`

rplr

Purpose

Estimate general input-output models using recursive pseudolinear regression method

Syntax

```
thm = rplr(z,nn,adm,adg)
[thm,yhat,P,phi] = rplr(z,nn,adm,adg,th0,P0,phi0)
```

Description

This is a direct alternative to `rpem` and has essentially the same syntax. See `rpem` for an explanation of the input and output arguments.

`rplr` differs from `rpem` in that it uses another gradient approximation. See Section 11.5 in Ljung (1999). Several of the special cases are well-known algorithms.

When applied to ARMAX models, (`nn = [na nb nc 0 0 nk]`), `rplr` gives the extended least squares (ELS) method. When applied to the output-error structure (`nn = [0 nb 0 0 nf nk]`), the method is known as HARF in the `adm = 'ff'` case and SHARF in the `adm = 'ng'` case.

`rplr` can also be applied to the time-series case in which an ARMA model is estimated with

```
z = y
nn = [na nc]
```

You can thus use `rplr` as an alternative to `rarmax` for this case.

See Also

`pem`, `rarmax`, `rarx`, `rbj`, `roe`, `rpem`

Purpose

Segment data and estimate models for each segment

Syntax

```
segm = segment(z,nn)
[segm,V,thm,R2e] = segment(z,nn,R2,q,R1,M,th0,P0,ll,mu)
```

Description

`segment` builds models of AR, ARX, or ARMAX/ARMA type,

$$A(q)y(t) = B(q)u(t-nk) + C(q)e(t)$$

assuming that the model parameters are piecewise constant over time. It results in a model that has split the data record into segments over which the model remains constant. The function models signals and systems that might undergo abrupt changes.

The input-output data is contained in `z`, which is either an `iddata` object or a matrix `z = [y u]` where `y` and `u` are column vectors. If the system has several inputs, `u` has the corresponding number of columns.

The argument `nn` defines the model order. For the ARMAX model

```
nn = [na nb nc nk]
```

where `na`, `nb`, and `nc` are the orders of the corresponding polynomials. See "Polynomial Representation of Transfer Functions" on page 3-11. Moreover, `nk` is the delay. If the model has several inputs, `nb` and `nk` are row vectors, giving the orders and delays for each input.

For an ARX model (`nc = 0`) enter

```
nn = [na nb nk]
```

For an ARMA model of a time series

```
z = y
nn = [na nc]
```

and for an AR model

```
nn = na
```

The output argument `segm` is a matrix, whose `k` row contains the parameters corresponding to time `k`. This is analogous to the output argument `thm` in `rarx` and `rarmax`. The output argument `thm` of `segment` contains the corresponding model parameters that have not yet been segmented. That is, they are not

piecewise constant, and therefore correspond to the outputs of the functions `rarmax` and `rarx`. In fact, `segment` is an alternative to these two algorithms, and has a better capability to deal with time variations that might be abrupt.

The output argument `V` contains the sum of the squared prediction errors of the segmented model. It is a measure of how successful the segmentation has been.

The input argument `R2` is the assumed variance of the innovations $e(t)$ in the model. The default value of `R2`, `R2 = []`, is that it is estimated. Then the output argument `R2e` is a vector whose `k`th element contains the estimate of `R2` at time `k`.

The argument `q` is the probability that the model undergoes at an abrupt change at any given time. The default value is `0.01`.

`R1` is the assumed covariance matrix of the parameter jumps when they occur. The default value is the identity matrix with dimension equal to the number of estimated parameters.

`M` is the number of parallel models used in the algorithm (see below). Its default value is `5`.

`th0` is the initial value of the parameters. Its default is zero. `P0` is the initial covariance matrix of the parameters. The default is 10 times the identity matrix.

`ll` is the guaranteed life of each of the models. That is, any created candidate model is not abolished until after at least ll time steps. The default is `ll = 1`. `Mu` is a forgetting parameter that is used in the scheme that estimates `R2`. The default is `0.97`.

The most critical parameter for you to choose is `R2`. It is usually more robust to have a reasonable guess of `R2` than to estimate it. Typically, you need to try different values of `R2` and evaluate the results. (See the example below.) `sqrt(R2)` corresponds to a change in the value $y(t)$ that is normal, giving no indication that the system or the input might have changed.

Algorithm

The algorithm is based on `M` parallel models, each recursively estimated by an algorithm of Kalman filter type. Each is updated independently, and its posterior probability is computed. The time-varying estimate `thm` is formed by weighting together the `M` different models with weights equal to their posterior probability. At each time step the model (among those that have lived at least 11 samples) that has the lowest posterior probability is abolished. A new model is started, assuming that the system parameters have jumped, with probability `q`, a random jump from the most likely among the models. The covariance matrix of the parameter change is set to `R1`.

After all the data are examined, the surviving model with the highest posterior probability is tracked back and the time instances where it jumped are marked. This defines the different segments of the data. (If no models had been abolished in the algorithm, this would have been the maximum likelihood estimates of the jump instances.) The segmented model `segm` is then formed by smoothing the parameter estimate, assuming that the jump instances are correct. In other words, the last estimate over a segment is chosen to represent the whole segment.

Examples

Check how the algorithm segments a sinusoid into segments of constant levels. Then use a very simple model $y(t) = b_1 * 1$, where `1` is a fake input and b_1 describes the piecewise constant level of the signal $y(t)$ (which is simulated as a sinusoid).

```
y = sin([1:50]/3)';
thm = segment([y,ones(size(y))],[0 1 1],0.1);
plot([thm,y])
```

By trying various values of `R2` (`0.1` in the above example), more levels are created as `R2` decreases.

Purpose

Select model order (structure)

Syntax

```
nn = selstruc(v)
[nn,vmod] = selstruc(v,c)
```

Description

`selstruc` is a function to help choose a model structure (order) from the information contained in the matrix `v` obtained as the output from `arxstruc` or `ivstruc`.

The default value of `c` is `'plot'`. The plot shows the percentage of the output variance that is not explained by the model as a function of the number of parameters used. Each value shows the best fit for that number of parameters. By clicking in the plot you can examine which orders are of interest. When you click `'Select'`, the variable `nn` is returned in the workspace as the optimal model structure for your choice of number of parameters. Several choices can be made.

`c = 'aic'` gives no plots, but returns in `nn` the structure that minimizes Akaike's Information Criterion (AIC),

$$V_{mod} = V\left(1 + \frac{2d}{N}\right)$$

where V is the loss function, d is the total number of parameters in the structure in question, and N is the number of data points used for the estimation. See `aic` for more details.

`c = 'mdl'` returns in `nn` the structure that minimizes Rissanen's Minimum Description Length (MDL) criterion.

$$V_{mod} = V\left(1 + \frac{d\log(N)}{N}\right)$$

When `c` equals a numerical value, the structure that minimizes

$$V_{mod} = V\left(1 + \frac{cd}{N}\right)$$

is selected.

The output argument `vmod` has the same format as `v`, but it contains the logarithms of the accordingly modified criteria.

Examples

```
V = arxstruc(data(1:200),data(201:400),struc(1:10,1:10,1:10))
nn = selstruc(V,0); %best fit to validation data
m = arx(data,nn)
```

set

Purpose

Set properties of models and `iddata` sets

Syntax

```
set(m,'Property',Value)
set(m,'Property1',Value1,...'PropertyN',ValueN)
set(m,'Property')
set(m)
```

Description

`set` is used to set or modify the properties of any of the objects in the toolbox (`iddata`, `idmodel`, `idgrey`, `idarx`, `idpoly`, `idss`). See the corresponding reference pages for a complete list of properties.

`set(m,'Property',Value)` assigns the value `Value` to the property of the object `m` specified by the string `'Property'`. This string can be the full property name (for example, `'SSParameterization'`) or any unambiguous case-insensitive abbreviation (for example, `'ss'`).

`set(m,'Property1',Value1,...'PropertyN',ValueN)` sets multiple properties with a single statement. In certain cases this might be necessary, since the model `m` must, for example, have state-space matrices of consistent dimensions after each `set` statement.

`set(m,'Property')` displays admissible values for the property specified by `'Property'`.

`set(m)` displays all assignable values of `m` and their admissible values.

The same result is also obtained by subassignment.

```
m.Property = Value
```

Purpose

Set matrix structure for `idss` objects

Syntax

```
setstruc(M,As,Bs,Cs,Ds.Ks,XOs)
setstruc(M,Mods)
```

Description

```
setstruc(M,As,Bs,Cs,Ds.Ks,XOs)
```

is the same as

```
set(M,'As',As,'Bs',Bs,'Cs',Cs,'Ds',Ds,'Ks',Ks,'XOs',XOs)
```

Use empty matrices for structure matrices that should not be changed. You can omit trailing arguments.

In the alternative syntax, `Mods` is a structure with fieldnames `As`, `Bs`, etc., with the corresponding values of the fields.

See Also

`idss`

setpname

Purpose

Set mnemonic parameter names for black-box model structures

Syntax

```
model = setpname(model)
```

Description

`model` is an `idmodel` object of `idarx`, `idpoly`, `idproc`, or `idss` type. The returned model has the `'PName'` property set to a cell array of strings that correspond to the symbols used in this manual to describe the parameters.

For single-input `idpoly` models, the parameters are called `'a1', 'a2', ...,'fn'`, as defined in "Polynomial Representation of Transfer Functions" on page 3-11.

For multiinput `idpoly` models, the *b* and *f* parameters have the output/input channel number in parentheses, as in `'b1(1,2)'`, `'f3(1,2)'`, etc.

For `idarx` models, the parameter names are as in `'-A(ky,ku)'` for the negative value of the *ky*-*ku* entry of the matrix in (Equation 3-50) and similarly for the *B* parameters.

For `idss` models, the parameters are named for the matrix entries they represent, such as `'A(4,5)'`, `'K(2,3)'`, etc.

For `idproc` models, the parameter names are as described under `idproc`.

This function is particularly useful when certain parameters are to be fixed. See the property `FixedParameter` under `Algorithm Properties`.

Purpose

Simulate linear models with confidence regions

Syntax

```
y = sim(m,u)
y = sim(m,u,'noise')
[y, ysd] = sim(m,u,'InitialState',init)
```

Description

`m` is an arbitrary `idmodel` object.

`u` is an `iddata` object, containing inputs only. (Any outputs are ignored). Both time-domain and frequency-domain signals are supported. The number of input channels in `u` must either be equal to the number of inputs of the model `m` or equal to the sum of the number of inputs and noise sources (number of outputs). In the latter case the last inputs in `u` are regarded as noise sources and a noise-corrupted simulation is obtained. The noise is scaled according to the property `m.NoiseVariance` in `m`. To obtain the right noise level according to the model, the noise inputs should be white noise with zero mean and unit covariance matrix. A simpler way of obtaining a noise-corrupted simulation with Gaussian noise is to add the argument `` `noise'. `` If no noise sources are contained in `u`, a noise-free simulation is obtained. `sim` applies both to time-domain and frequency-domain `iddata` objects, but no standard deviations are obtained for frequency-domain signals.

`sim` returns `y`, containing the simulated output, as an `iddata` object.

`init` gives access to the initial states:

- `init = 'm'` (default) uses the internally stored initial state of model `m`.
- `init = 'z'` uses zero initial state.
- `init = x0`, where `x0` is a column vector of appropriate length, uses this value as the initial state. For multi-experiment inputs, `x0` has as many columns as there are experiments to allow for different initial conditions. Notice that for a continuous-time model `m`, `x0` is the initial state for this model. Any modifications of the initial state that sampling might require are automatically handled. If `m` has a non-zero `InputDelay`, and you need to access the values of the inputs during this delay, you must first apply `inpd2nk(m)`. If `m` is a continuous-time model, it must first be sampled before `inpd2nk` can be applied.

The second output argument `ysd` is the standard deviation of the simulated output. This is not available for frequency-domain data.

u can also be given as a matrix with the number of columns being either the number of inputs in m or the sum of the number of inputs and outputs. Then y and ysd are returned as matrices. Continuous-time models, however, require u to be given as iddata.

If m is a continuous-time model, it is first converted to discrete time with the sampling interval given by ue, taking into account the intersample behavior of the input (ue.InterSample). See "Discrete- and Continuous-Time Models" on page 3-68.

Examples

Simulate a given system m0 (for example, created by idpoly).

```
e = iddata([],randn(500,1));
u = iddata([],idinput(500,'prbs'));
y = sim(m0,[u e]);
z = [y u]; % An iddata object with y as output and u as input.
```

The same result is obtained by

```
u = iddata([],idinput(500,'prbs'));
y = sim(m0,u,'noise');
z = [ y u];
```

or

```
u = idinput(500,'prbs');
y = sim(m0,u,'noise');
z = iddata(y,u);
```

Validate a model by comparing a measured output y with one simulated using an estimated model m.

```
yh = sim(m,u);
plot(y,yh)
```

See Also

iddata, idpoly, idarx, idss, idgrey, simsd

Purpose

Simulate models with uncertainty using Monte Carlo method

Syntax

```
simsd(m,u)
simsd(m,u,N,'noise',Ky)
[y,ysd] = simsd(m,u)
```

Description

`u` is an `iddata` object containing the inputs. `m` is a model given as any `idmodel` object. `N` random models are created according to the covariance information given in `m`. The responses of each of these models to the input `u` are computed and graphed in the same diagram. If the argument `'noise'` is included, noise is added to the simulation in accordance with the noise model of `m` and its own uncertainty. `Ky` denotes the output numbers to be plotted. (The default is `all`).

The default value is `N=10`.

With output arguments

```
[y,ysd] = simsd(m,u)
```

No plots are produced, but `y` is returned as a cell array with the simulated outputs, and `ysd` is the estimated standard deviation of `y`, based on the `N` different simulations. If `u` is an `iddata` object, so are the contents of the cells of `y` and `ysd`; otherwise, they are returned as vectors/matrices. In the `iddata` case,

```
plot(y{:})
```

thus plots all the responses.

`sim` and `simsd` have similar syntaxes. Note that `simsd` computes the standard deviation by Monte Carlo simulation, while `sim` uses differential approximations (the Gauss approximation formula). They might give different results.

Examples

Plot the step response of the model `m` and evaluate how it varies in view of the model's uncertainty.

```
step1 = [zeros(5,1); ones(20,1)];
simsd(m,step1)
```

See Also

`sim`

size

Purpose

Dimensions of `iddata`, `idmodel`, and `idfrd` objects

Syntax

```
d = size(m)
[ny,nu,Npar,Nx] = size(model)
[N, ny, nu, Nexp] = size(data)
ny = size(data,2)
ny = size(data,'ny')
size(model)
size(idfrd_object)
```

Description

`size` describes the dimensions of `iddata`, `idmodel`, and `idfrd` objects.

iddata

For `iddata` objects, the sizes returned are, in this order,

- `N` = the length of the data record. For multiple-experiment data, `N` is a row vector with as many entries as there are experiments.
- `ny` = the number of output channels.
- `ny` = the number of input channels.
- `Ne` = the number of experiments.

To access just one of these sizes, use `size(data,k)` for the `k`th dimension or `size(data,'N')`, `size(data,'ny')`, etc.

When called with one output argument, `d = size(data)` returns

- `d = [N ny nu]` if the number of experiments is `1`.
- `d = [sum(N) ny nu Ne]` if the number of experiments is `Ne > 1`.

idmodel

For `idmodel` objects the sizes returned are, in this order,

- `ny` = the number of output channels.
- `nu` = the number of input channels.
- `Npar` = the length of the `ParameterVector` (number of estimated parameters).
- `Nx` = the number of states for `idss` and `idgrey` models.

In this case the individual dimensions are obtained by `size(mod,2)`, `size(mod,'Npar')`, etc.

When `size` is called with one output argument, `d = size(mod)`, `d` is given by

```
[ny nu Npar]
```

idfrd

For `idfrd` models, the sizes returned are, in this order,

- `ny` = the number of output channels.
- `nu` = the number of input channels.
- `Nf` = the number of frequencies.
- `Ns` = the number of spectrum channels.

In this case the individual dimensions are obtained by `size(mod,2)`, `size(mod,'Nf')`, etc.

When `size` is called with one output argument, `d = size(fre)`, `d` is given by

```
[ny nu Nf Ns]
```

When `size` is called with no output arguments, in any of these cases, the information is displayed in the MATLAB Command Window.

Purpose

Estimate frequency response and spectrum using spectral analysis

Syntax

```
g = spa(data)
g = spa(data,M,w,maxsize)
[g,phi,spe] = spa(data)
```

Description

spa estimates the transfer function g and the noise spectrum Φ_v of the general linear model

$$y(t) = G(q)u(t) + v(t)$$

where $\Phi_v(\omega)$ is the spectrum of $v(t)$.

data contains the output-input data as an iddata object. The data can be complex valued. data can be both time domain and frequency domain. data can also be an idfrd object.

g is returned as an idfrd object (see idfrd) with the estimate of $G(e^{i\omega})$ at the frequencies ω specified by row vector w. The default value of w is

```
w = [1:128]/128*pi/Ts
```

Here Ts is the sampling interval of data.

g also includes information about the spectrum estimate of $\Phi_v(\omega)$ at the same frequencies. Both outputs are returned with estimated covariances, included in g. See idfrd.

M is the length of the lag window used in the calculations. The default value is

```
M = min(30,length(data)/10)
```

Changing the value of M controls the frequency resolution of the estimate. The resolution corresponding to M is approximately π/M rad/sampling interval. The value of M exchanges bias for variance in the spectral estimate. As M is increased, the estimated functions show more detail, but are more corrupted by noise. The sharper peaks a true frequency function has, the higher M it needs. See etfe as an alternative for narrowband signals and systems. The function spafdr allows the frequency resolution to depend on the frequency. See also “Estimating Spectra and Frequency Functions” on page 3-15.

maxsize controls the memory-speed tradeoff (see Algorithm Properties).

For time series, where `data` contains no input channels, `g` is returned with the estimated output spectrum and its estimated standard deviation.

When `spa` is called with two or three output arguments,

- `g` is returned as an `idfrd` model with just the estimated frequency response from input to output and its uncertainty.
- `phi` is returned as an `idfrd` model, containing just the spectrum data for the output spectrum $\Phi_v(\omega)$ and its uncertainty.
- `spe` is returned as an `idfrd` model containing spectrum data for all output-input channels in `data`. That is, if `z = [data.OutputData, data.InputData]`, `spe` contains as spectrum data the matrix-valued power spectrum of `z`.

$$S = \sum_{m=-M}^{M} Ez(t+m)z(t)'\exp(-iWmT)win(m)$$

Here $win(m)$ is weight at lag `m` of an M-size Hamming window and W is the frequency value i rad/s. Note that `'` denotes complex-conjugate transpose.

The normalization of the spectrum differs from the one used by `spectrum` in the Signal Processing Toolbox. See "Spectrum Normalization and the Sampling Interval" on page 3-107 for a more precise definition.

Examples

With default frequencies,

```
g = spa(z);
bode(g)
```

With logarithmically spaced frequencies,

```
w = logspace(-2,pi,128);
g= spa(z,[],w); % (empty matrix gives default)
bode(g,'sd',3)
bode(g('noise'),'sd',3) % The noise spectrum with confidence
interval of 3 standard deviations.
```

Algorithm

The covariance function estimates are computed using `covf`. These are multiplied by a Hamming window of lag size `M` and then transformed using a Fourier transform. The relevant ratios and differences are then formed. For the default frequencies, this is done using a fast Fourier transform, which is more efficient than for user-defined frequencies. For multivariable systems, a straightforward `for` loop is used.

Note that `M` = γ is in Table 6.1 of Ljung (1999). The standard deviations are computed as on pages 184 and 188 in Ljung (1999).

See Also

`bode`, `etfe`, `idfrd`, `nyquist`, `spafdr`

Purpose

Estimate frequency response and spectrum using spectral analysis with frequency-dependent resolution

Syntax

```
g = spafdr(data)
g = spafdr(data,Resol,w)
```

Description

`spafdr` estimates the transfer function `g` and the noise spectrum Φ_v of the general linear model

$$y(t) = G(q)u(t) + v(t)$$

where $\Phi_v(\omega)$ is the spectrum of $v(t)$.

`data` contains the output-input data as an `iddata` object. The data can be complex valued, and either time or frequency domain. It can also be an `idfrd` object containing frequency-response data.

`g` is returned as an `idfrd` object (see `idfrd`) with the estimate of $G(e^{i\omega})$ at the frequencies ω specified by row vector `w`. `g` also includes information about the spectrum estimate of $\Phi_v(\omega)$ at the same frequencies. Both results are returned with estimated covariances, included in `g`. See `idfrd`. The normalization of the spectrum is the same as described under `spa`.

Frequencies

The frequency variable `w` is either specified as a row vector of frequencies, or as a cell array `{wmin,wmax}`. In the latter case the covered frequencies will be 50 logarithmically spaced points from `wmin` to `wmax`. You can change the number of points to `NP` by entering `{wmin,wmax,NP}`.

Omitting `w` or entering it as an empty matrix gives the default value, which is 100 logarithmically spaced frequencies between the smallest and largest frequency in data. For time-domain data, this means from `1/N*Ts` to `pi*Ts`, where `Ts` is the sampling interval of data and `N` is the number of data.

Resolution

The argument `Resol` defines the frequency resolution of the estimates. The resolution (measured in rad/s) is the size of the smallest detail in the frequency function and the spectrum that is resolved by the estimate. The resolution is a tradeoff between obtaining estimates with fine, reliable details, and suffering from spurious, random effects: The finer the resolution, the higher the variance

in the estimate. `Resol` can be entered as a scalar (measured in rad/s), which defines the resolution over the whole frequency interval. It can also be entered as a row vector of the same length as `w`. Then `Resol(k)` is the local, frequency-dependent resolution around frequency `w(k)`.

The default value of `Resol`, obtained by omitting it or entering it as the empty matrix, is `Resol(k) = 2(w(k+1)-w(k))`, adjusted upwards, so that a reasonable estimate is guaranteed. In all cases, the resolution is returned in the variable `g.EstimationInfo.WindowSize`.

Algorithm

If the data is given in the time domain, it is first converted to the frequency domain. Then averages of `Y(w)Conj(U(w))` and `U(w)Conj(U(w))` are formed over the frequency ranges `w`, corresponding to the desired resolution around the frequency in question. The ratio of these averages is then formed for the frequency-function estimate, and corresponding expressions define the noise spectrum estimate.

See Also

`bode`, `etfe`, `idfrd`, `nyquist`, `spa`

Purpose

Convert `idmodel` objects of System Identification Toolbox to LTI models of Control System Toolbox

Syntax

```
sys = ss(mod)
sys = ss(mod,'m')
```

Description

`mod` is any `idmodel` object: `idgrey`, `idarx`, `idpoly`, `idproc`, `idss`, or `idmodel`.

`sys` is returned as an `ss` LTI model object. The noise input channels in `mod` are treated as follows: consider a model `mod` with both measured input channels u (nu channels) and noise channels e (ny channels) with covariance matrix Λ

$$y = Gu + HLv$$
$$\mathrm{cov}(v) = I$$

Both measured input channels u and normalized noise input channels v in `mod` are input channels in `sys`. The noise input channels belong to the `InputGroup 'Noise'`, while the others belong to the `InputGroup 'Measured'`. The names of the noise input channels are `v@yname`, where `yname` is the name of the corresponding output channel. This means that the LTI object realizes the transfer function [G HL].

To transform only the measured input channels in `sys`, use

```
sys = ss(mod('m')) or sys = ss(mod,'m')
```

This gives a representation of G only.

For a time series, (no measured input channels, nu = 0), the LTI representations in `ss` contains the transfer functions from the normalized noise sources v to the outputs, that is, HL. If the model `mod` has both measured and noise inputs, `sys = ss(mod('n'))` gives a representation of the additive noise.

In addition, the normal subreferencing can be used.

```
sys = ss(mod(1,[3 4]))
```

If you want to describe [G H] or H (unnormalized noise), from e to y, first use

```
mod = noisecnv(mod)
```

to convert the noise channels e to regular input channels. These channels are assigned the names `e@yname`.

See Also frd, tf, zpk

Purpose

Convert model to state-space form

Syntax

```
[A,B,C,D,K,X0] = ssdata(m)
[A,B,C,D,K,X0,dA,dB,dC,dD,dK,dX0] = ssdata(m)
```

Description

`m` is the model given as any `idmodel` object. `A`, `B`, `C`, `D`, `K`, and `X0` are the matrices in the state-space description

$$\tilde{x}(t) = Ax(t) + Bu(t) + Ke(t)$$

$$x(0) = x0$$

$$y(t) = Cx(t) + Dx(t) + e(t)$$

where $\tilde{x}(t)$ is $\dot{x}(t)$ or $x(t + Ts)$ depending on whether `m` is a continuous-time or discrete-time model.

`dA`, `dB`, `dC`, `dD`, `dK`, and `dX0` are the standard deviations of the state-space matrices.

If the underlying model itself is a state-space model, the matrices correspond to the same basis. If the underlying model is an input-output model, an observer canonical form representation is obtained.

For a time-series model (no measured input channels, u = `[]`), B and D are returned as the empty matrices.

Subreferencing models in the usual way (see `idmodel` properties) will give the state-space representation of the chosen channels. Notice in particular that

```
[A,B,C,D] = ssdata(m('m'))
```

gives the response from the measured inputs. This is a model without a disturbance description. Moreover,

```
[A,B,C,D,K] = ssdata(m('n'))
```

(`'n'` as in “noise”) gives the disturbance description, that is, a time-series description of the additive noise with no measured inputs, so that `B = []` and `D = []`.

To obtain state-space descriptions that treat all input channels, both u and e, as measured inputs, first apply the conversion

```
m = noisecnv(m)
```

or

```
m = noisecnv(m,'norm')
```

where the latter case first normalizes e to v, where v has a unit covariance matrix. See the reference page for `noisecnv`.

Algorithm

The computation of the standard deviations in the input-output case assumes that an A polynomial is not used together with an F or D polynomial in (Equation 3-43). For the computation of standard deviations in the case that the state-space parameters are complicated functions of the parameters, the Gauss approximation formula is used together with numerical derivatives. The step sizes for this differentiation are determined by `nuderst`.

See Also

`idmodel`, `idss`, `nuderst`

Purpose

Plot step response with confidence regions

Syntax

```
step(m)
step(data)
step(m,'sd',sd,Time)
step(data,'sd',sd,'PW',na,Time)
step(m1,m2,...,dat1, ...,mN,Time,'sd',sd)
step(m1,'PlotStyle1',m2,'PlotStyle2',...,dat1,'PlotStylek',...,mN,
   'PlotStyleN',Time,'sd',sd)
[y,t,ysd] = step(m)
mod = step(data)
```

Description

`step` can be applied both to `idmodels` and to `iddata` sets, as well as to any mixture.

For a discrete-time `idmodel` `m`, the step response `y` and, when required, its estimated standard deviation `ysd`, are computed using `sim`. When called with output arguments, `y`, `ysd`, and the time vector `t` are returned. When `step` is called without output arguments, a plot of the step response is shown. If `sd` is given a value larger than zero, a confidence region around the response is drawn. It corresponds to the confidence of `sd` standard deviations. If the input argument list contains `'fill'`, this region is plotted as a filled area.

Setting the Time Interval

The start time `T1` and the end time `T2` can be specified by `Time = [T1 T2]`. If `T1` is not given, it is set to `-T2/4`. The negative time lags (the step is always assumed to occur at time `0`) show possible feedback effects in the data when the step is estimated directly from data. If `Time` is not specified, a default value is used.

Estimating the Step Response from data

For an `iddata` set data, `step(data)` estimates a high-order, noncausal FIR model after first having prefiltered the data so that the input is “as white as possible.” The step response of this FIR model and, when asked for, its confidence region, are then plotted. Note that it might not be possible always to deliver the demanded time interval in this case, because of lack of excitation in the data. A warning is then issued. When called with an output argument, `step`, in the `iddata` case, returns this FIR model, stored as an `idarx` model. The

order of the prewhitening filter can be specified as `na`. The default value is `na = 10`.

Several Models/Data Sets

Any number and any mixture of models and data sets can be used as input arguments. The responses are plotted with each input/output channel (as defined by the models and data sets `InputName` and `OutputName`) as a separate plot. Colors, line styles, and marks can be defined by `PlotStyle` values, as in

```
step(m1,'b-*',m2,'y--',m3,'g')
```

Noise Channels

The noise input channels in `m` are treated as follows: Consider a model `m` with both measured input channels u (nu channels) and noise channels e (ny channels) with covariance matrix Λ

$$y = Gu + He$$
$$\mathrm{cov}(e) = \Lambda = LL'$$

where L is a lower triangular matrix. Note that `m.NoiseVariance` = Λ. The model can also be described with a unit variance, normalized noise source v:

$$y = Gu + HLv$$
$$\mathrm{cov}(v) = I$$

- `step(m)` plots the step response of the transfer function G.
- `step(m('n'))` plots the step response of the transfer function H (ny inputs and ny outputs).The input channels have names `e@yname`, where `yname` is the name of the corresponding output.
- If `m` is a time series, that is, nu = `0`, `step(m)` plots the step response of the transfer function H.
- `step(noisecnv(m))` plots the step response of the transfer function [G H] (nu+ny inputs and ny outputs). The noise input channels have names `e@yname`, where `yname` is the name of the corresponding output.
- `step(noisecnv(m,'norm'))` plots the step response of the transfer function [G HL] (nu+ny inputs and ny outputs). The noise input channels have names `v@yname`, where `yname` is the name of the corresponding output.

Arguments

If `step` is called with a single `idmodel` `m`, the output argument `y` is a 3-D array of dimension `Nt`-by-`ny`-by-`nu`. Here `Nt` is the length of the time vector `t`, `ny` is the

number of output channels, and `nu` is the number of input channels. Thus `y(:,ky,ku)` is the response in the `ky`th output channel to a step in the `ku`th input channel. No plot is produced when output arguments are used.

`ysd` has the same dimensions as *`y`* and contains the standard deviations of `y`. This is normally computed using `sim`. However, when the model `m` contains an estimated delay (dead time) as in certain process models, the standard deviation is estimated with Monte Carlo techniques, using `simsd`.

If `step` is called with an output argument and a single data set in the input arguments, the output is returned as an `idarx` model `mod` containing the high-order FIR model, and its uncertainty. By calling `step` with `mod`, the responses can be displayed and returned without your having to redo the estimation.

Examples

`step(data,'sd',3)` estimates and plots the step response

```
mod = step(data)
step(mod,'sd',3)
```

See Also

`cra`, `impulse`

struc

Purpose

Generate model structure matrices

Syntax

```
NN = struc(NA,NB,NK)
```

Description

`struc` returns in `NN` the set of model structures composed of all combinations of the orders and delays given in row vectors `NA`, `NB`, and `NK`. The format of `NN` is consistent with the input format used by `arxstruc` and `ivstruc`. The command is intended for single-input systems only.

Examples

The statement

```
NN = struc(1:2,1:2,4:5);
```

produces

```
NN =
 1  1  4
 1  1  5
 1  2  4
 1  2  5
 2  1  4
 2  1  5
 2  2  5
```

See Also

`arxstruc`, `ivstruc`, `selstruc`

Purpose

Convert `idmodel` objects of System Identification Toolbox to transfer-function LTI models of Control System Toolbox

Syntax

```
sys = tf(mod)
sys = tf(mod,'m')
```

Description

`mod` is any `idmodel` object: `idgrey`, `idarx`, `idpoly`, `idproc`, `idss`, or `idmodel`.

`sys` is returned as ta `tf` LTI model object. The noise input channels in `mod` are treated as follows:

Consider a model `mod` with both measured input channels u (nu channels) and noise channels e (ny channels) with covariance matrix Λ

$$y = Gu + He$$
$$\mathrm{cov}(e) = \Lambda = LL'$$

where L is a lower triangular matrix. Notice that `mod.NoiseVariance` = Λ . The model can also be described with a unit variance, normalized noise source v.

$$y = Gu + HLv$$
$$\mathrm{cov}(v) = I$$

Both measured input channels u and normalized noise input channels v in `mod` are input channels in `sys`. The noise input channels belongs to the `InputGroup 'Noise'`, while the others belong to the `InputGroup 'Measured'`. The names of the noise input channels will be `v@yname`, where `yname` is the name of the corresponding output channel. This means that the LTI object realizes the transfer function [G HL].

To transform only the measured input channels in `mod`, use

```
sys = tf(mod('m')) or sys = tf(mod,'m')
```

This gives a representation of G only.

For a time series, (no measured input channels, nu = `0`), the LTI representation contains the transfer functions from the normalized noise sources v to the outputs, that is, HL. If the model `mod` has both measured and noise inputs, `sys = tf(mod('n'))` gives a representation of the additive noise.

In addition, you can use normal subreferencing.

```
sys = tf(mod(1,[3 4]))
```

If you want to describe [*G H*] or *H* (unnormalized noise), from *e* to *y*, first use

```
mod = noisecnv(mod)
```

to convert the noise channels *e* to regular input channels. These channels are assigned the names `e@yname`.

See Also

`frd`, `ss`, `zpk`

Purpose

Convert model to transfer-function form

Syntax

```
[num,den] = tfdata(m)
[num,den,sdnum,sdden] = tfdata(m)
[num,den,sdnum,sdden] = tfdata(m,'v')
```

Description

`m` is a model given as any `idmodel` object with `ny` output channels and `nu` input channels.

`num` is a cell array of dimension `ny`-by-`nu`. `num{ky,ku}` (note the curly brackets) contains the numerator of the transfer function from input `ku` to output `ky`. This numerator is a row vector whose interpretation is described below.

Similarly, `den` is an `ny`-by-`nu` cell array of the denominators.

`sdnum` and `sdden` have the same formats as `num` and `den`. They contain the standard deviations of the numerator and denominator coefficients.

If `m` is a SISO model, adding an extra input argument `'v'` (for vector) will return `num` and `den` as vectors rather than cell arrays.

The formats of `num` and `den` are the same as those used by the Signal Processing Toolbox and the Control System Toolbox, both for continuous-time and discrete-time models. See "Examining Models" on page 3-57 and the examples below.

The noise input channels in `m` are treated as follows: Consider a model `m` with both measured input channels u (nu channels) and noise channels e (ny channels) with covariance matrix Λ

$$y = Gu + He$$
$$\mathrm{cov}(e) = \Lambda = LL'$$

where L is a lower triangular matrix. Note that `m.NoiseVariance` = Λ. The model can also be described with a unit variance, normalized noise source v:

$$y = Gu + HLv$$
$$\mathrm{cov}(v) = I$$

- `tfdata(m)` returns the transfer function G.
- `tfdata(m('n'))` returns the transfer function H (ny inputs and ny outputs).

- If `m` is a time series, that is, *nu* = `0`, `tfdata(m)` returns the transfer function *H*.
- `tfdata(noisecnv(m))` returns the transfer function [*G H*] (*nu*+*ny* inputs and *ny* outputs).
- `tfdata(noisecnv(m,'norm'))` returns the transfer function [*G HL*] (*nu*+*ny* inputs and *ny* outputs).

Examples

For a continuous-time model,

```
num = [1 2]
den = [1 3 0]
```

corresponds to the transfer function

$$G(s) = \frac{s+2}{s^2+3s}$$

For a discrete-time model,

```
num = [2 4 0]
den = [1 2 3 5]
```

corresponds to the transfer function

$$H(z) = \frac{2z^2+4z}{z^3+2z^2+3z+5}$$

which is the same as

$$H(q) = \frac{2q^{-1}+4q^{-2}}{1+2q^{-1}+3q^{-2}+5q^{-3}}$$

Note that for discrete-time models, `idpoly` and `polydata` have a different interpretation of the numerator vector, in case it does not have the same length as the denominator vector. To avoid confusion, fill out with zeros to make numerator and denominator vectors the same length. Do this with `tfdata`.

See Also

`idpoly`, `noisecnv`

Purpose

Return date and time when object was created or last modified

Syntax

```
timestamp(obj)
ts = timestamp(obj)
```

Description

`obj` is any `idmodel`, `iddata`, or `idfrd` object. `timestamp` returns or displays a string with information about when the object was created and last modified.

Purpose

Plot model characteristics using LTI viewer in Control System Toolbox

Syntax

```
view(m)
view(m('n'))
view(m1,...,mN,Plottype)
view(m1,PlotStyle1,...,mN,PlotStyleN)
```

Description

`m` is the output-input data to be graphed, given as any `idfrd` or `idmodel` object. After appropriate model transformations, the LTI viewer of the Control System Toolbox is invoked. This allows `bode`, `nyquist`, `impulse`, `step`, and `zero/poles` plots.

To compare several models `m1,...,mN`, use `view(m1,...,mN)`. With `PlotStyle`, the color, line style, and marker of each model can be specified.

```
view(m1,'y:*',m2,'b')
```

Adding `Plottype` as a last argument specifies the type of plot in which `view` is initialized. `Plottype` is any of `'impulse'`, `'step'`, `'bode'`, `'nyquist'`, `'nichols'`, `'sigma'`, or `'pzmap'`. It can also be given as a cell array containing any collection of these strings (up to 6) in which case a multiplot is shown.

`view` does not display confidence regions. For that, use `bode`, `nyquist`, `impulse`, `step`, and `pzmap`.

The noise input channels in `m` are treated as follows: Consider a model `m` with both measured input channels u (nu channels) and noise channels e (ny channels) with covariance matrix Λ

$$y = Gu + He$$
$$cov(e) = \Lambda = LL^T$$

where L is a lower triangular matrix. Note that `m.NoiseVariance` = Λ. The model can also be described with a unit variance, normalized noise source v:

$$y = Gu + HLv$$
$$cov(v) = I$$

- `view(m)` plots the characteristics of the transfer function G.

- `view(m('n'))` plots the characteristics of the transfer function *HL* (*ny* inputs and *ny* outputs). The input channels have names `v@yname`, where `yname` is the name of the corresponding output.
- If `m` is a time series, that is, *nu* = `0`, `view(m)` plots the characteristics of the transfer function *HL*.
- `view(noisecnv(m))` plots the characteristics of the transfer function [*G H*] (*nu*+*ny* inputs and *ny* outputs). The noise input channels have names `e@yname`, where `yname` is the name of the corresponding output.
- `view(noisecnv(m,'norm'))` plots the characteristics of the transfer function [*G HL*] (*nu*+*ny* inputs and *ny* outputs). The noise input channels have names `v@yname`, where `yname` is the name of the corresponding output.

`view` does not give access to all of the features of `ltiview`. Use

```
ml = ss(m), ltiview(Plottype,ml,...)
```

to reach these options.

See Also

`bode`, `impulse`, `nyquist`, `step`, `pzmap`

zpk

Purpose

Convert `idmodel` objects of System Identification Toolbox to state-space LTI models of Control System Toolbox

Syntax

```
sys = zpk(mod)
sys = zpk(mod,'m')
```

Description

`mod` is any `idmodel` object: `idgrey`, `idarx`, `idpoly`, `idproc`, `idss`, or `idmodel`.

`sys` is returned as a `zpk` LTI model object. The noise input channels in `mod` are treated as follows: consider a model `mod` with both measured input channels u (nu channels) and noise channels e (ny channels) with covariance matrix Λ

$$y = Gu + He$$
$$\mathrm{cov}(e) = \Lambda = LL'$$

where L is a lower triangular matrix. Note that `mod.NoiseVariance` = Λ. The model can also be described with a unit variance, normalized noise source v.

$$y = Gu + HLv$$
$$\mathrm{cov}(v) = I$$

Both measured input channels u and normalized noise input channels v in `mod` are input channels in `sys`. The noise input channels belongs to the `InputGroup 'Noise'`, while the others belong to the `InputGroup 'Measured'`. The names of the noise input channels are given by `v@yname`, where `yname` is the name of the corresponding output channel. This means that the LTI object realizes the transfer function [G HL].

To transform only the measured input channels in `sys`, use

```
sys = zpk(mod('m')) or sys = zpk(mod,'m')
```

This gives a representation of G only.

For a time series, (no measured input channels, nu = 0), the LTI representation contains the transfer functions from the normalized noise sources v to the outputs, that is, HL. If the model `mod` has both measured and noise inputs, `sys = zpk(mod('n'))` gives a representation of the additive noise.

In addition, the normal subreferencing can be used.

```
sys = zpk(mod(1,[3 4]))
```

If you want to describe [*G H*] or *H* (unnormalized noise), from *e* to *y*, first use

```
mod = noisecnv(mod)
```

to convert the noise channels *e* to regular input channels. These channels are assigned have the names `e@yname`.

See Also

`frd`, `ss`, `tf`

zpkdata

Purpose

Compute zeros, poles, and transfer-function gains of models

Syntax

```
[z,p,k] = zpkdata(m)
[z,p,k,dz,dp,dk] = zpkdata(m)
[z,p,k,dz,dp,dk] = zpkdata(m,'v')
```

Description

`m` is a model given as any `idmodel` object with `ny` output channels and `nu` input channels.

`z` is a a cell array of dimension `ny`-by-`nu`. `z{ky,ku}` (note the curly brackets) contains the zeros of the transfer function from input `ku` to output `ky`. This is a column vector of possibly complex numbers.

Similarly, `p` is an `ny`-by-`nu` cell array containing the poles.

`k` is a `ny`-by-`nu` matrix whose `ky`-`ku` entry is the transfer function gain of the transfer function from input `ku` to output `ky`. Note that the transfer function gain is the value of the leading coefficient of the numerator when the leading coefficient of the denominator is normalized to 1. It thus differs from the static gain. The static gain can be retrieved as `Ks = freqresp(m,0)`.

`dz` contains the covariance matrices of the zeros in the following way: `dz` is a `ny`-by-`nu` cell array. `dz{ky,ku}` contains the covariance information about the zeros of the transfer function from `ku` to `ky`. It is a 3-D array of dimension 2-by-2-by-`Nz`, where `Nz` is the number of zeros. `dz{ky,ku}(:,:,kz)` is the covariance matrix of the zero `z{ky,ku}(kz)`, so that the 1-1 element is the variance of the real part, the 2-2 element is the variance of the imaginary part, and the 1-2 and 2-1 elements contain the covariance between the real and imaginary parts.

`dp` contains the covariance matrices of the poles in the same way.

`dk` is a matrix containing the variances of the elements of `k`.

If `m` is a SISO model, adding an extra input argument `'v'` (for vector) returns `z` and `p` as vectors rather than cell arrays.

Note that the zeros and the poles are associated with the different channel combinations. To obtain the so-called transmission zeros, use `tzero`.

The noise input channels in `m` are treated as follows: Consider a model `m` with both measured input channels u (nu channels) and noise channels e (ny channels) with covariance matrix Λ

$$y = Gu + He$$
$$\mathrm{cov}(e) = \Lambda = LL'$$

where L is a lower triangular matrix. Note that `m.NoiseVariance` = Λ. The model can also be described with a unit variance, normalized noise source v.

$$y = Gu + HLv$$
$$\mathrm{cov}(v) = I$$

Then,

- `zpkdata(m)` returns the zeros and poles of G.
- `zpkdata(m('n'))` returns the zeros and poles of H (ny inputs and ny outputs).
- If `m` is a time series, that is, nu = `0`, `zpkdata(m)` returns the zeros and poles of H.
- `zpkdata(noisecnv(m))` returns the zeros and poles of the transfer function [G H] (nu+ny inputs and ny outputs).
- `zpkdata(noisecnv(m,'norm'))` returns the zeros and poles of the transfer function [G HL] (nu+ny inputs and ny outputs).

The procedure handles both models in continuous and discrete time.

Note that you cannot rely on information about zeros and poles at the origin and at infinity for discrete-time models. (This is a somewhat confusing issue anyway.)

Algorithm

The poles and zeros are computed using `ss2zp`. The covariance information is computed using the Gauss approximation formula, using the parameter covariance matrix contained in `m`. When the transfer function depends on the parameters, numerical differentiation is applied. The step sizes for the differentiation are determined in the M-file `nuderst`.

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